

KEY

PHYSICS 241

TEST 1

Monday, February 17, 2003

This 15-question test (each question is worth approximately 6.67 points) is worth 100 points, each question is weighted equally. Please fill out the answer sheet with soft lead pencil. Be sure to give your name, student ID #, date, Course #, Test 1, and ****SIGN**** the answer sheet. Be prepared to present your Student picture ID card when handing in your answer sheet. You may keep the sheets with the questions and your work.

Pick the nearest value for your answer (there may be slight round-off errors). If your answer is significantly different from all possible answers, you have made some mistake.

Don't get hung up too long over any one question until you have tried all of them.

You are expected to bring your own sheet of equations and words explaining the equations. Here are a few possibly useful constants or equations.

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \quad k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 = 1/4\pi\epsilon_0$$

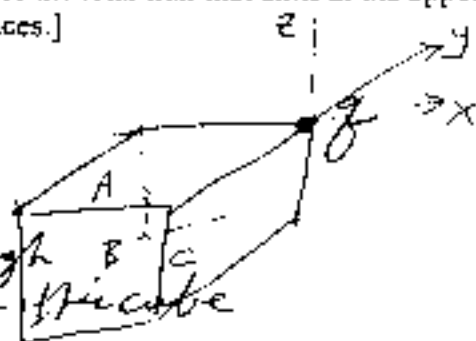
$$g = 9.8 \text{ m/s}^2 \quad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$\text{weight} = mg$$

- 1.) A point charge q is placed exactly at the corner of a cube. What is the electric flux through one of the "opposite" cube faces which does NOT touch the charge? [Hint: do some of the faces pass zero flux? Hint: exploit the symmetries of this configuration to (a) determine the fraction of the total flux that aims at the opposite faces and (b) share the flux among those faces.]

- a) q/ϵ_0
 b) $q/4\pi\epsilon_0$
 c) $q/6\epsilon_0$
 d) $q/12\epsilon_0$
 e) $q/24\epsilon_0$

8 "octants"
 for $x+ve$
 $y+ve$
 $z+ve$



all flux goes through only one octant for this cube

$\frac{1}{3}$ of this flux goes through each of faces A, B, C

$\therefore \frac{1}{24}$ through one of these "opposite" faces

$$\Phi_{total} = \frac{q}{\epsilon_0}, \quad \Phi_A = \frac{q}{24\epsilon_0} \text{ etc.}$$

- 2.) The flux of the uniform electric field $E = (13\hat{i} + 30\hat{j} + 16\hat{k}) \text{ N/C}$ through a 2.0 m^2 portion of the yz plane is

- a) $26 \text{ N m}^2/\text{C}$
 b) $32 \text{ N m}^2/\text{C}$
 c) $92 \text{ N m}^2/\text{C}$
 d) $48 \text{ N m}^2/\text{C}$
 e) $60 \text{ N m}^2/\text{C}$

only the \hat{i} portion projects onto the normal to the yz plane, which points along \hat{i} so:

$$\hat{i} \cdot \hat{i} = +1$$

$$\Phi = A_{yz} \cdot E_x = 2 \text{ m}^2 \times 13 \frac{\text{N}}{\text{C}} = 26 \text{ N m}^2/\text{C}$$

3.) Charges $Q_1 = +10\text{nC}$ and $Q_2 = -30\text{nC}$ are placed on the x axis at $x = 0$ and $x = 50\text{ cm}$, respectively. What is the net electric flux, in Nm^2/C , through a spherical surface of radius $r = 40\text{ cm}$ centered on the origin of coordinates?

- a) 4520
- b) -2260
- c) 90
- d) 1130
- e) -180

only $+10\text{ nC}$ is inside the sphere

$$\phi = \frac{10 \cdot 10^{-9}\text{ C}}{\epsilon_0} = \frac{10 \cdot 10^{-9}\text{ C}}{8.85 \cdot 10^{-12} (\text{Nm}^2/\text{C}^2)^{-1}}$$

$$= 1.13 \times 10^3 \frac{\text{Nm}^2}{\text{C}} \phi$$

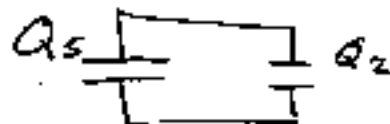
4.) A $50\ \mu\text{F}$ capacitor is charged to 150 V . It is then connected to both ends of a $20\ \mu\text{F}$ capacitor. How much charge is stored on the positive plate of the $20\ \mu\text{F}$ capacitor?

- a) 2.14 mC
- b) 5.36 mC
- c) 7.50 mC
- d) 3.00 mC
- e) 100. mC

$$Q_0 = CV$$

$$= 50 \cdot 10^{-6}\text{ F} \times 150\text{ V}$$

$$Q_0 = 7.5 \cdot 10^{-3}\text{ C}$$



After connecting

V is different after connection, Q_5 is on the big cap.
 Q_2 is on the small cap

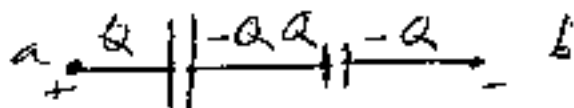
$\& Q_0 = Q_2 + Q_5$ is conserved

$$\text{So } \left. \begin{array}{l} Q_2 = C_2 V' \\ Q_5 = C_5 V' \\ Q_0 = (C_2 + C_5) V' \end{array} \right\} \Rightarrow Q_2 = \frac{C_2}{C_2 + C_5} Q_0 = \frac{20}{20 + 50} Q_0 = \frac{2}{7} Q_0$$

$$Q_2 = \frac{2}{7} (7.5) \cdot 10^{-3}\text{ C} = \underline{\underline{2.14\text{ mC}}}$$

5.) A $50 \mu\text{F}$ capacitor and a $20 \mu\text{F}$ capacitor are connected in series and charged to 150 V . How much charge is stored on the positive plate of the $20 \mu\text{F}$ capacitor?

- a) 2.14 mC
- b) 5.36 mC
- c) 7.50 mC
- d) 3.00 mC
- e) $100. \text{ mC}$



$$V_{ab} = 150 \text{ V} \quad C_{ab} = \frac{1}{\frac{1}{20} + \frac{1}{50}} = \frac{20 \times 50}{20 + 50} \mu\text{F}$$

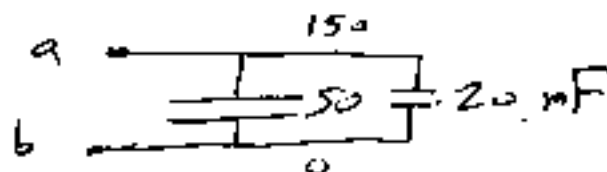
$$Q = C_{\text{series}} \cdot V =$$

$$Q = \frac{20}{7} (50 \mu\text{F}) \cdot 150 \text{ V} = 2.14 \text{ mC} \quad !$$

Same as before - cute, no?

6.) A $50 \mu\text{F}$ capacitor and a $20 \mu\text{F}$ capacitor are connected in parallel and charged to 150 V . How much charge is stored on the positive plate of the $20 \mu\text{F}$ capacitor?

- a) 2.14 mC
- b) 5.36 mC
- c) 7.50 mC
- d) 3.00 mC
- e) $100. \text{ mC}$

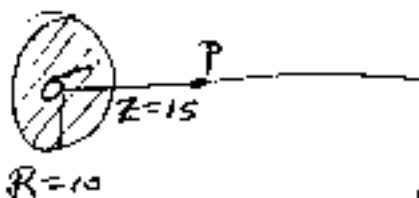


both see 150 V

$$Q_{20} = C_{20} V = 20 \mu\text{F} (150 \text{ V}) = 3000 \mu\text{C} = 3 \text{ mC}$$

- 7.) A circular disk of uniform charge density $\sigma = 120 \mu\text{C}/\text{m}^2$ has a radius of 10 cm. What is the electric field magnitude, in N/C, at a point on the axis of symmetry 15 cm from the disk?

- a) 0.97×10^3
 b) 3.18×10^4
 c) 5.32×10^5
 d) 9.11×10^5
 e) 1.14×10^6



$$\left(\frac{1}{2\epsilon_0}\right) = 2\pi k$$

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right) = 2\pi \frac{9.0 \times 10^9 \text{ Nm}^2}{\text{C}^2} \left(120 \frac{\mu\text{C}}{\text{m}^2}\right) \left(\right)$$

$$\left(1 - \frac{.15}{\sqrt{.1^2 + .15^2}}\right) = 18\pi (120)(10^3) \frac{\text{N}}{\text{C}} \cdot (1 - .832)$$

$$E = 6.78 \cdot 10^6 \frac{\text{N}}{\text{C}} (.168) = 1.14 \cdot 10^6 \text{ N/C}$$

- 8.) Find the magnitude of the electric field, E , in V/m, at the point (1.5m, -1.5m) in the x - y plane for the potential function $V(x, y) = (x^2y - y^2x - 10)V$, using the gradient operators.

- a) 7.75
 b) 9.55
 c) 15.50
 d) 0
 e) 77.5

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{E} = - (2xy - y^2, x^2 - 2xy, 0) \text{ V/m}$$

$$\vec{E} = -2 (1.5)(-1.5) + 1.5^2, -1.5^2 + 2(1.5)(-1.5), 0)$$

$$\vec{E} = (2.25(2+1), 2.25(-1-2), 0)$$

$$\vec{E} = 2.25(3, -3, 0)$$

$$|\vec{E}| = 2.25 \times 3 \times \sqrt{2} = 9.546 \text{ V/m}$$

- 9.) An active thundercloud can have a potential exceeding 3 MV relative to the Earth's surface. Assume a not untypical cloud system whose bottom is 1 mile (~1.6 km) above the earth and has an area of 100 km². Estimate the energy, U, stored in this cloud if it is at the stated potential (treat it and the Earth as a giant capacitor.)

- a) 0.83 MJ
 b) 2.49 MJ
 c) 4.98 MJ
 d) 8.30 MJ
 e) 9.96 MJ

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \cdot 10^{-12} \cdot 100 (1000\text{m})^2}{1.6 \cdot 10^3 \text{m}}$$

$$C = 5.53 \cdot 10^{-7} \text{ F}$$

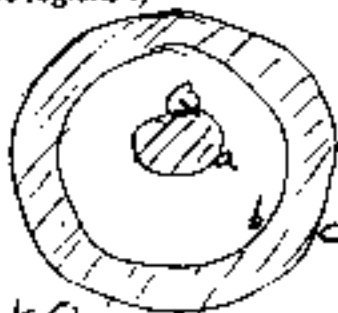
$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} (5.53 \cdot 10^{-7} \text{ F}) (3 \cdot 10^6 \text{ V})^2$$

$$U = 2.489 \cdot 10^6 \text{ J} = 2.49 \text{ MJ}$$

- 10.) An uncharged conducting spherical shell with inner and outer radii 1.8 m and 2.1 m surrounds a concentric conducting solid sphere of 0.3 m radius charged with +13 mC. What is the potential of the inner sphere at a distance of 0.1 m from the center? (relative to $V = 0$ at $r = \infty$). [Hint: notice carefully ALL equipotentials, and do two separate integrals in the two empty-space regions!]

- a) 3.25 V
 b) 117 MV
 c) 234 MV
 d) 314 MV
 e) 380 MV

$$\left. \begin{array}{l} \Delta V_{bc} = 0 \\ \Delta V_{ca} = 0 \end{array} \right\} \text{Conductors}$$



$d = \infty$

$$\Delta V_{cd} = \int_{\infty}^c \frac{kQ}{r^2} = kQ \left(\frac{1}{2.1\text{m}} - \frac{1}{\infty} \right)$$

$$\Delta V_{ab} = \int_b^a \frac{kQ}{r^2} = kQ \left(\frac{1}{0.3} - \frac{1}{1.8} \right)$$

$$kQ = 9 \cdot 10^9 (13 \cdot 10^{-3} \text{ C})$$

$$\Delta V = \Delta V_{cd} + \Delta V_{ab} = 1.17 \cdot 10^8 \left(\frac{1}{2.1} + \frac{1}{0.3} - \frac{1}{1.8} \right)$$

$$\Delta V = 3.807 \cdot 10^8 \text{ V} \approx 3.80 \cdot 10^2 \text{ MV}$$

for any $r \leq a$

$$K = 9.0 \times 10^9 = \frac{1}{4\pi\epsilon_0} = \frac{1}{4\pi(8.85 \times 10^{-12})}$$

11.) The above system is filled with oil (dielectric constant $\kappa = 27$). What is its capacitance (between the shell and the center sphere)?

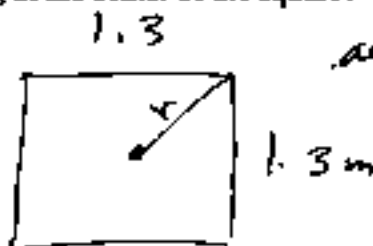
- a) 40 pF
- b) 40 nF
- c) 1.08 pF
- d) 1.08 nF
- e) 2.7 nF

$$C = \kappa \cdot 4\pi\epsilon_0 \frac{ab}{b-a} = \frac{27}{9.0 \times 10^9} \left(\frac{0.3(1.8)}{1.8-0.3} \right)$$

$$C = \frac{27}{9.0 \times 10^9} = \frac{1.8}{5} = 1.08 \times 10^{-9} \text{ F}$$

12.) Charges of 20, 40, 60, and -80 mC are placed at the corners of a 1.3 m square. What is the electric potential, V, at the center of the square?

- a) 1.96 GV
- b) 1.17 GV
- c) 0.78 GV
- d) 0.39 GV
- e) 0.28 GV



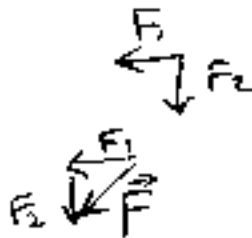
all four $r = \text{same}$
 $r = 1.3 \text{ m} / \sqrt{2}$

$$V = \sum_{i=1}^4 \frac{kQ_i}{r} = \frac{9.0 \times 10^9 \text{ Nm}^2/\text{C}^2}{0.919 \text{ m}} (20 + 40 + 60 - 80) \times 10^{-3} \text{ C}$$

$$V = +3.916 \times 10^8 \left[\frac{\text{Nm}}{\text{C}} \text{ or } \frac{\text{J}}{\text{C}} \text{ or } \text{V} \right]$$

13.) A $2 \mu\text{C}$ charge is placed at the origin, an identical charge is placed 2m from the origin on the x axis, and a third identical charge is placed 2 m from the origin on the y axis. The magnitude of the force on the charge at the origin is:

- a) $9.0 \times 10^{-3} \text{ N}$
- b) $6.4 \times 10^{-3} \text{ N}$
- c) $1.3 \times 10^{-2} \text{ N}$
- d) $1.8 \times 10^{-2} \text{ N}$
- e) $3.6 \times 10^{-2} \text{ N}$



$$|F_{1+2}| = \frac{k q_1 q_2}{r^2} = \frac{9 \times 10^9 (2 \times 10^{-6} \text{ C})^2}{(2 \text{ m})^2}$$

$$|F_{1+2}| = 9 \times 10^{-3} = 9 \times 10^{-3}$$

$$|\vec{F}| = |\vec{F}_1 + \vec{F}_2| = \sqrt{2} (|F_1|) = 12.7 \times 10^{-3} \text{ N}$$

14.) The force exerted by a uniform electric field on a dipole is:

- a) parallel to the dipole moment
- b) perpendicular to the dipole moment
- c) parallel to the electric field
- d) perpendicular to the electric field
- e) none of the above

i.e. zero

IT'S zero

Torques only,
 dipole has equal $+q$ & $-q$, just
 separated. \vec{F}_+ & \vec{F}_- are \parallel to each other
 & cancel translationally

Torque $\neq 0$ due to (possible) lever arm,
 depending on dipole orientation

- 15.) A charged oil drop with a mass of 2×10^{-4} kg is held suspended by a downward electric field of 300 N/C. The charge on the drop is [warning: be careful about signs]

a) $+1.5 \times 10^{-6}$ C

b) -1.5×10^{-6} C

c) $+6.5 \times 10^{-6}$ C

d) -6.5×10^{-6} C

e) 0

↑ +g
↓ g

$$\vec{F}_g = -mg = m\vec{g} = mg(-\hat{g})$$

$$= -2.0 \times 10^{-4} \text{ kg} (9.8 \text{ m/s}^2)$$

$$= -1.96 \times 10^{-3} \text{ N (down)}$$

to balance this) need level up and

$$\vec{F}_{em} = q\vec{E} \text{ upward but } \vec{E} = 300 \text{ N/C } (-\hat{g}) \text{ (down)}$$

$$\text{So } q = \frac{\vec{F}_{em}}{\vec{E}} = \frac{+1.96 \times 10^{-3} \text{ N } (+\hat{g}) \text{ (up)}}{-300 \text{ N/C } (-\hat{g}) \text{ (down)}}$$

$$q = -6.5 \times 10^{-6} \text{ C}$$

