Key

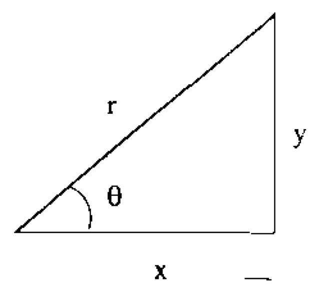
Physics 241 – Exam #1

February 21 2006

This exam consists of 13 problems on 9 pages. Please check that you have them all. Each problem is worth 12 points except #13 which is worth 6 points. The maximum score possible is 150.

All of the formulas that you will need are given below. You may also use a calculator.

$\sin\theta = y/r$ $\qquad$ $\cos\theta = x/r$ $\qquad$ $\tan\theta = y/x$



$e = 1.6 \times 10^{-19}$ C $\qquad$ $k = 9.0 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$ $\qquad$ $\epsilon_0 = 8.9 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$

$$F = \frac{kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\epsilon_0 r^2} \qquad E = \frac{kq}{r^2} \qquad \Phi = \int \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \qquad \text{charged plane}: E = \frac{\sigma}{2\epsilon_0}$$

$$\Delta V = \frac{\Delta U_E}{q} = -\int \vec{E}\cdot d\vec{l} \qquad dV = -\vec{E}\cdot d\vec{l} \qquad \text{point charge}: V = \frac{kq}{r} \qquad U_E = q_0V = \frac{kqq_0}{r}$$

$$E_x = -\frac{\partial V}{\partial x} \qquad 1\ \text{Volt} = 1\ \text{J / C} \qquad 1\ \text{Volt/m} = 1\ \text{N / C} \qquad U_E = \frac{1}{2}qV \qquad C = \frac{q}{V}$$

$$\text{Surface area(sphere)} = 4\pi R^2 \qquad \text{capacitor}: U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV = \frac{1}{2}CV^2 \qquad u_E = \frac{1}{2}\epsilon_0 E^2$$

$$\text{parallel plate capacitor}: C = \frac{\epsilon_0 A}{d} \qquad \text{isolated spherical capacitor}: C = 4\pi\epsilon_0 R$$

$$\text{capacitors in parallel}: C = C_1 + C_2 + C_3 ... \qquad \text{capacitors in series}: \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ...$$

$$C = \kappa C_0 \qquad I = \frac{\Delta q}{\Delta t} \qquad R = \frac{V}{I} \qquad R = \rho \frac{L}{A} \qquad V = IR$$

$$P = IV = I^2 R = \frac{V^2}{R} \qquad P = \mathcal{E} I \qquad \text{resistors in series} : R = R_1 + R_2 + R_3 \ldots$$

$$\text{resistors in parallel} : \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots \qquad q(t) = q_0 e^{-t/(RC)} = q_0 e^{-t/\tau}$$

$$I(t) = \frac{V}{R} e^{-t/(RC)} = I_0 e^{-t/\tau} \qquad \tau = RC$$

$$q(t) = C\mathcal{E}(1 - e^{-t/(RC)}) = q_0(1 - e^{-t/\tau}) \qquad I(t) = \frac{\mathcal{E}}{R} e^{-t/(RC)} = I_0 e^{-t/\tau}$$

1. Six electrons are placed on a metal sphere. If the potential of the sphere is −35 V, what is the radius of the sphere?

(a) 4.6×10^{-21} m

(b) 9.0×10^{-9} m

(c) 4.1×10^{-9} m

(d) 2.5×10^{-10} m

(e) 5.0×10^{-10} m

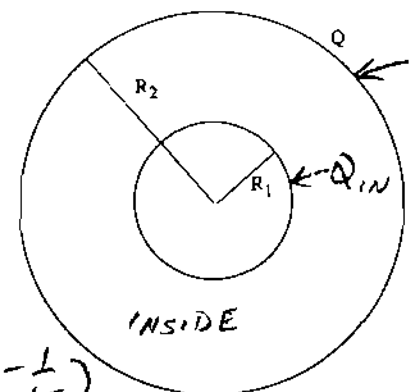
$$V = \frac{kq}{r}$$

$$r = \frac{kq}{V} = \frac{(9.0\times10^{9})(6)(-1.6\times10^{-19})}{-35}$$

$$= 2.5 \times 10^{-10}\ \text{m}$$

2. Two thin insulating shells have radii $R_1 = 0.50$ m and $R_2 = 1.5$ m. Both shells are charged uniformly: the outer shell has a total charge +3.5 C, while the inner shell carries an unknown amount of charge. If the electric field outside the bigger shell is zero, what is the electric potential difference between the two shells?

To make $E_{outside} = 0$



$+3.5C = Q$

$$Q_{IN} = -Q$$

$$E_{INSIDE} = \frac{kQ_{in}}{r^2}$$

$$\Delta V = \frac{kQ_{IN}}{R_1} - \frac{kQ_{IN}}{R_2} = k(-Q)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 9\times10^{9}(-3.5)\left(\frac{1}{0.5} - \frac{1}{1.5}\right)$$

$$= 4.2 \times 10^{10}\ \text{V}$$

~~(a) 6.3×10^{10} V with the inner shell at a lower potential~~

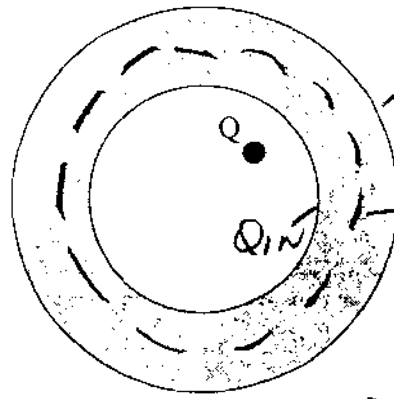
(b) 4.2×10^{10} V with the inner shell at a lower potential

(c) 8.4×10^{10} V with the inner shell at a lower potential

(d) 3.2×10^{10} V with the inner shell at a lower potential

(e) 6.3×10^{10} V with the inner shell at a higher potential

3. A hollow metal sphere has a total charge of zero. A point charge Q sits inside the hollow region, slightly away from the center of the sphere as shown below. What is the charge induced on the outer surface of the metal sphere?



Q_{out}

$E = 0$

So $\phi = 0$ for this surface

$\Rightarrow Q_{IN} = -Q$

$\boxed{\Rightarrow Q_{out} = -Q_{in} = +Q}$

(a) $+Q$
(b) $-Q$
(c) $+Q/2$
(d) $-2Q$
(e) zero

4. Two infinite uniformly charged planes are separated by a distance L. The plane on the left has a charge per unit area of -3σ, and is at $V = 0$. The plane on the right has a charge per unit area $+\sigma$. What is the potential of the plane on the right?

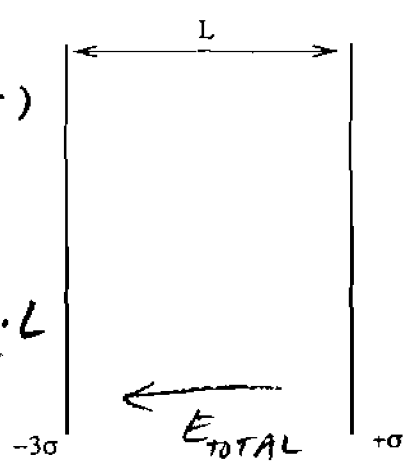
For one plane

$E = \frac{\sigma}{2\epsilon_0}$

Between planes $E_{TOTAL} = \frac{\sigma}{2\epsilon_0} - \frac{(-3\sigma)}{2\epsilon_0}$

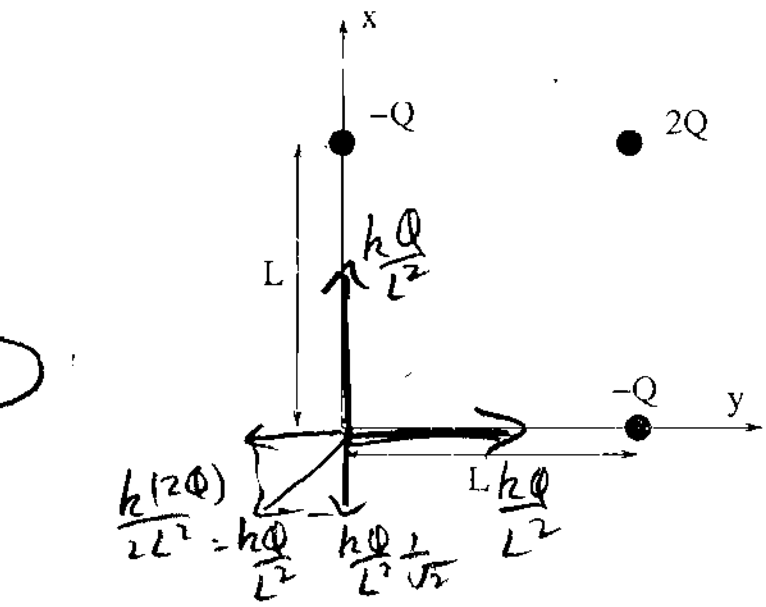
$= \frac{2\sigma}{\epsilon_0}$

$\Delta V = E_{TOTAL} \cdot L$



(a) $4\sigma L/\epsilon_0$
(b) $\sigma L^2/\epsilon_0$
(c) $-2\sigma L/\epsilon_0$
(d) $2\sigma L/\epsilon_0$
(e) $\sigma L/(2\epsilon_0)$

5. Three point charges are arranged as shown below. What is the electric field at the origin?



(a) $-0.71kQ/L^2$ along x
$-0.71kQ/L^2$ along y

(b) $+0.29kQ/L^2$ along x
$+0.29kQ/L^2$ along y

(c) $+0.50kQ/L^2$ along x
$+0.50kQ/L^2$ along y

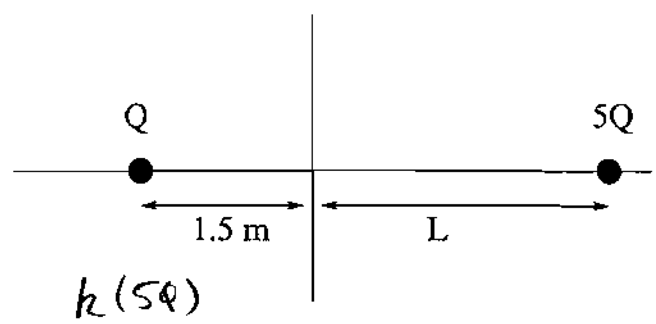
(d) $+kQ/L^2$ along x
$+kQ/L^2$ along y

(e) $+(0.71)kQ/L^2$ along x
$+(0.71)kQ/L^2$ along y

x: $E_x = +\frac{kQ}{L^2}\left(+1 - \frac{1}{\sqrt{2}}\right) = +0.29\frac{kQ}{L^2}$

y: $E_y =$

6. A point charge Q is located as shown on the x axis a distance of 1.5 m from the origin. A second point charge $5Q$ is located as shown an unknown distance L from the origin. The electric field at the origin is zero. Find L.



(a) $L = 2.3$ m

(b) $L = 4.5$ m

(c) $L = 7.5$ m

(d) $L = 1.5$ m

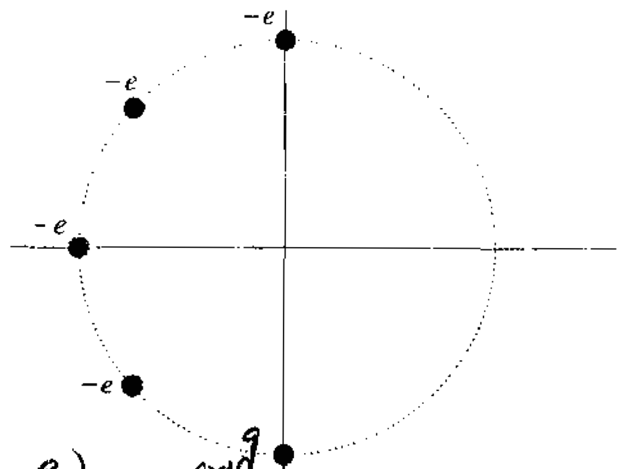
(e) $L = 3.4$ m

$\frac{kQ}{(1.5)^2} = \frac{k(5Q)}{L^2}$

$L^2 = (1.5)^2\,\frac{5Q}{Q}$

$L = (1.5)\sqrt{5} = 3.4$ m

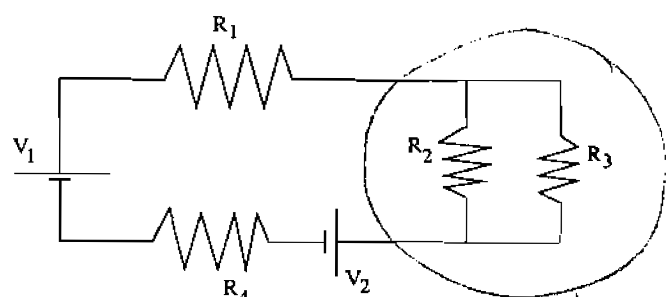
7. Five electrons are equally spaced around a semicircle of radius $R = 0.15$ m as shown. What is the electric potential at the origin?



(a) -3.2×10^{-7} V
(b) $+3.2 \times 10^{-8}$ V
(c) -4.8×10^{-8} V
(d) -9.6×10^{-8} V
(e) zero

$$V = \sum \frac{kQ}{r} = \frac{5k(-e)}{R} = \frac{9\times10^{9} \cdot 5(-1.6\times10^{-19})}{0.15} = -4.8\times10^{-8}\ \text{V}$$

8. Consider the circuit shown below, with $R_1 = 1.0$ kΩ, $R_2 = 1.5$ kΩ, $R_3 = 2.5$ kΩ, $R_4 = 2.0$ kΩ, $V_1 = 5.5$ V and $V_2 = 3.5$ V. What is the current through resistor R_4?



$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$V_1 - IR_1 - IR' - IR_4 + V_2 = 0$$

$$R' = 0.94\ \text{k}\Omega$$

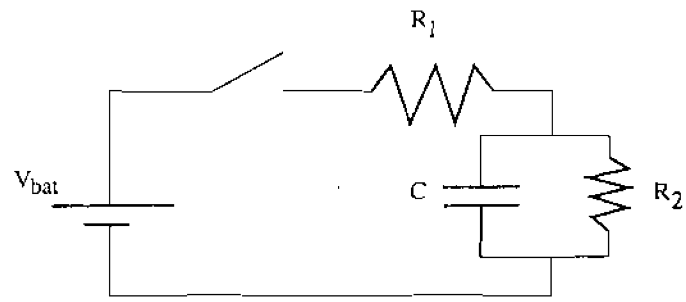
(a) 0.29 mA
(b) 1.0 mA
(c) 0.44 mA
(d) 2.3 mA
(e) 0.51 mA

$$I = \frac{V_1 - V_2}{R_1 + R_4 + R'} = \frac{(5.5 - 3.5)\ \text{V}}{(1 + 2 + 0.94)\ \text{k}\Omega} = \frac{2.0}{3.94\times10^{3}} = 0.51\ \text{mA}$$

9. Consider the circuit shown below, with $R_1 = 3.0\ \text{k}\Omega$, $R_2 = 2.0\ \text{k}\Omega$, $C = 5.0\ \mu\text{F}$ and a battery voltage is $V_{\text{bat}} = 9.0\ \text{V}$. The switch is initially open for a very long time. The switch is then closed at $t = 0$. What is the current through resistor R_1 the instant after the switch is closed?

at $t=0$ $\quad q=0 \Rightarrow V_{cap}=0 \approx$ short circuit

$I = \frac{V_{bat}}{R_1} = \frac{9.0}{3.0\times10^3} = \underline{3.0\ mA}$



(a) 1.8 mA

(b) zero

(c) 7.5 mA

(d) 3.0 mA

(e) 9.0 mA

10. Consider again the circuit in Problem 9 above. After the switch is closed for a very long time, what is the charge on the capacitor?

(a) 1.8×10^{-5} C

(b) 3.7×10^{-5} C

(c) zero

(d) 2.7×10^{-5} C

(e) 4.5×10^{-5} C

$t=\infty \quad I_{cap}=0$

all current goes through R_2

$I = \frac{V_{bat}}{R_1+R_2} = \frac{9.0}{(3.0+2.0)k\Omega} = 1.8\ mA$

$V_{cap} = I \cdot R_2 = (1.8\times10^{-3})(2.0\ k\Omega) = 3.6\ V$

$q = C\, V_{cap} = 5\times10^{-6} \cdot 3.6 = \underline{1.8\times10^{-5}\ C}$

11. An electron moves a distance of 7.5 m through a region where the electric field E is constant and parallel to the displacement. The electron's potential energy increases by 9.5×10^{-20} J. What is the magnitude of E?

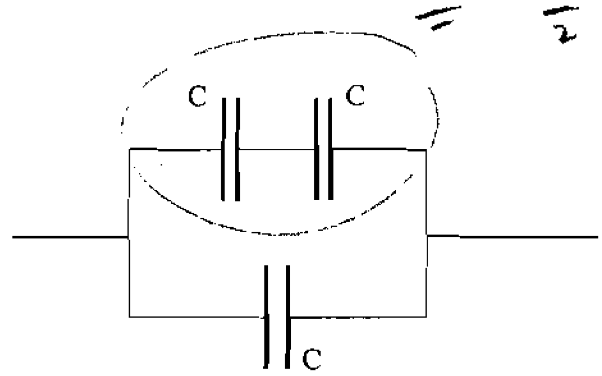
$q \cdot E \cdot L = \Delta U$

$|E| = \left|\frac{\Delta U}{qL}\right| = \frac{9.5\times10^{-20}}{1.6\times10^{-19}\cdot(7.5)} = \underline{0.079\ V/m}$

(a) 0.13 V/m
(b) 0.079 V/m
(c) 0.59 V/m
(d) 0.20 V/m
(e) 1.2×10^{-20} V/m

12. Three identical capacitors are connected as shown. If the equivalent capacitance of this combination is 37 μF, find C.

$= \frac{C}{2} =$



(a) 37 μF
(b) 19 μF
(c) 74 μF
(d) 12 μF
(e) 25 μF

$C + \frac{C}{2} = \frac{3C}{2} = 37\mu F$

$\underline{C = 25\mu F}$

Note: Problem 13 is worth 6 points.

13. In class we did a demonstration with a capacitor with movable plates and an air gap. We observed how the voltage across the capacitor varied as the plate separation d was changed. Which of the following statements best describes this experiment?

    (a) Inserting a dielectric between the plates increases the capacitance.

    (b) The voltage difference depends on the charge on the capacitor.

    (c) The RC time constant does not depend on the plate spacing.

    (d) The voltage decreases as the plate separation is decreased, because the capacitance gets bigger as d is made smaller.

    (e) Capacitors with movable plates are designed to torment students.

The End