Exam 2 — Physics 241

November 11, 2003

Solutions - Codrington

1. Please print your name on the top edge of the op-scan sheet.
2. Use a #2 pencil to fill in your full name, your student identification number, your recitation division number, and finally the answers for problems 1–12.
3. One (both sides) 8 1/2" x 11" crib sheet is allowed. It must be hand-written.

Useful equations and constants:

$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{\ell}\times\vec{r}}{r^3} \qquad \oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \qquad F = q\vec{v}\times\vec{B} \qquad F = i\vec{\ell}\times\vec{B}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad \mu = NiA \qquad \frac{F}{\ell} = \frac{\mu_0 i_1 i_2}{2\pi d} \qquad d\varepsilon = -Bvd\ell \qquad N\phi_B = Li$$

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\phi_B}{dt} \qquad \varepsilon = -N\frac{d\phi_B}{dt} \qquad \phi_B = BA$$

$$U_E = \frac{1}{2}CV^2 \qquad V = \varepsilon(1-e^{-t/RC}) \qquad i = \frac{\varepsilon}{R}e^{-t/RC} \qquad q = q_0 e^{-t/RC}$$

$$U_B = \frac{1}{2}Li^2 \qquad V = \varepsilon(1-e^{-Rt/L}) \qquad i = \frac{\varepsilon}{R}(1-e^{-Rt/L}) \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$\omega' = \sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} \qquad \varepsilon_m = i_m Z \qquad Z = \sqrt{R^2+(X_L-X_C)^2} \qquad X_L = \omega L \qquad X_C = \frac{1}{\omega C}$$

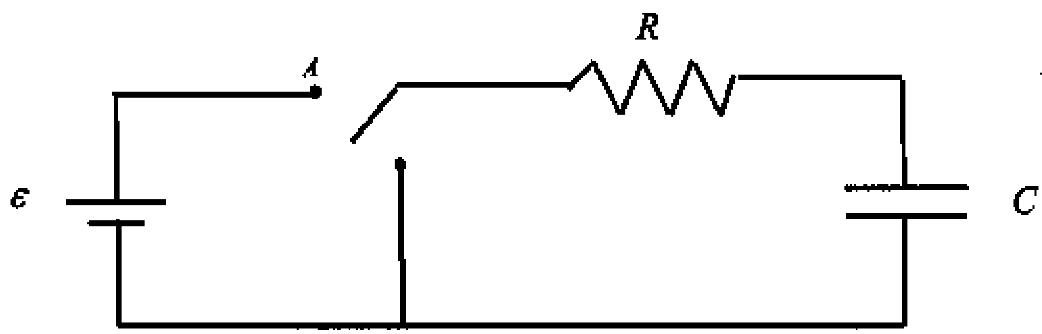
$$\tan\phi = \frac{X_L - X_C}{R} \qquad P_{av} = \frac{1}{2}\frac{(\varepsilon_m)^2}{Z}\cos\phi \qquad \bar{S} = \frac{1}{\mu_0 c}\frac{(E_m)^2}{2}$$

$$\mu_0 = 4\pi\times10^{-7}\,T\cdot m/A \qquad \varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N\cdot m^2}$$

$$e = 1.6\times10^{-19}C \qquad m_p = 1.67\times10^{-27}kg$$

$$\mu(micro)\Rightarrow 10^{-6} \qquad n(nano)\Rightarrow 10^{-9} \qquad p(pico)\Rightarrow 10^{-12}$$

1. A circuit for charging a capacitor is shown below. The switch is thrown to position A at t=0. The circuit has elements ε=12V, R=100 Ω and C=10μF. How long does it take for the capacitor to be charged to 99.9 percent of its final charge?



(a) 6.91 ms
(b) 0.99 ms
(c) 0.0001 ms
(d) 0.001 ms
(e) 0.0691 ms

Let charge on capacitor be $q(t)$
Let final charge on capacitor be q_f
Want t at which $q(t) = .999\, q_f$

$$\Rightarrow .999\, q_f = \underbrace{q(t) = q_f\left(1 - e^{-t/\tau_c}\right)}_{\text{equation for charging a capacitor}}$$

$$\Rightarrow .999 = 1 - e^{-t/\tau_c}$$

$$\Rightarrow e^{-t/\tau_c} = 1 - .999 = .001$$

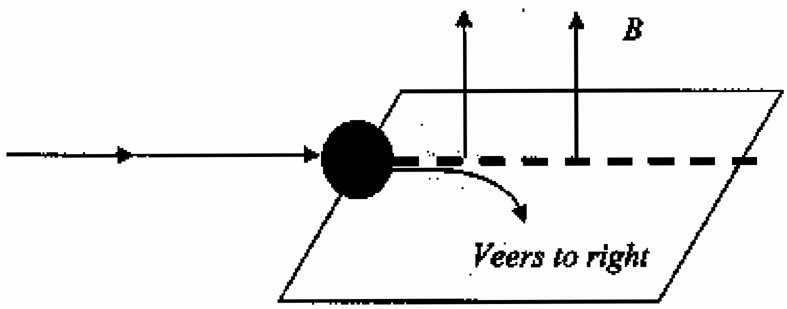
$$\Rightarrow -t/\tau_c = \ln(.001)$$

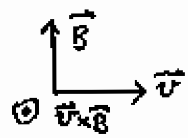
$$\Rightarrow t = -\underbrace{\tau_c}_{=RC} \ln(.001) = -RC\ln(.001)$$

$$= -(100\,\Omega)(10\times10^{-6}\,F)\ln(.001) = \boxed{6.91\times10^{-3}\ s}$$

$$\Rightarrow \boxed{A}$$

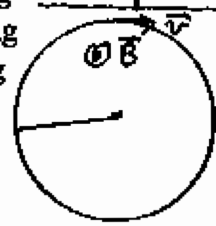
2. A particle of unknown charge q and unknown mass m moves at a speed $v=4.8\times10^6$ m/s in the $+x$-direction into a region of constant magnetic field. The field has magnitude B=0.5 T and is orientated in the $+y$-direction. The particle is deflected in the $+z$-direction and traces out a fragment of a circle of radius R=0.1m. What is the sign of the charge and what is its ratio of q/m?





(a) Neutral, q/m=0
(b) Negative, q/m=1.4 ×10⁻⁸ C/Kg
(c) Positive, q/m=1.4×10⁻⁸ C/Kg
(d) Negative, q/m=9.6×10⁷ C/Kg
(e) Positive, q/m=9.6×10⁷ C/Kg

$\left.\begin{array}{l}\vec{v}\times\vec{B} \text{ points out of page} \\ \vec{F} \text{ points out of page}\end{array}\right\} \Rightarrow$ in $\vec{F} = q\vec{v}\times\vec{B}$, q must be <u>positive</u>



For a particle moving at constant speed v in a circle of radius R, the centripetal acceleration is

$$a_c = \frac{v^2}{R}$$

Thus the centripetal force is

$$F_c = ma_c = \frac{mv^2}{R}$$

The only force acting on the particle is the magnetic force $\vec{F}_B$, so we must have

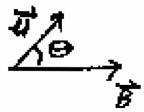
$$\underbrace{F_c}_{= \frac{mv^2}{R}} = |\vec{F}_B| = |q\vec{v}\times\vec{B}| = |q|\,|\vec{v}\times\vec{B}| = |q|\underbrace{|\vec{v}|}_{v}\underbrace{|\vec{B}|}_{B}\underbrace{\sin 90^\circ}_{1}$$

$$\Rightarrow \frac{mv^{\cancel{2}}}{R} = |q|\cancel{v}B \Rightarrow \frac{|q|}{m} = \frac{v}{RB} = \frac{4.8\times10^6\ \text{m/s}}{(.1\ \text{m})(.5\ \text{T})} = \boxed{9.6\times10^7\ \frac{C}{Kg}}$$

positive

$$\Rightarrow \boxed{E}$$

3. A loop of wire carrying a current has a magnetic dipole moment $\mu = 5\times10^{-4}$ A·m^2. Initially, the vector μ makes an angle of 90^0 with a 0.5 T magnetic field. As it turns to become aligned with the field, the **work done by the field** is:



Potential energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ as a function of the angle θ between $\vec{\mu}$ and $\vec{B}$ is

$$U(\theta) = -\vec{\mu}\cdot\vec{B} = -|\vec{\mu}||\vec{B}|\cos\theta$$

(a) 0 J
(b) 2.5×10^{-4} J
(c) -2.5×10^{-4} J
(d) 1.0×10^{-4} J
(e) -1.0×10^{-4} J

Change in potential energy is

$$\Delta U = U_f - U_i = U(0^\circ) - U(90^\circ) = (-\mu B \underbrace{\cos 0^\circ}) - (-\mu B \underbrace{\cos 90^\circ}_{0}) = -\mu B$$

Let $W_{\vec{B}\text{-field}}$ be the work done by the $\vec{B}$-field.

$$W_{\vec{B}\text{-field}} = \Delta KE = -\Delta PE = -\Delta U = -(-\mu B) = \mu B$$

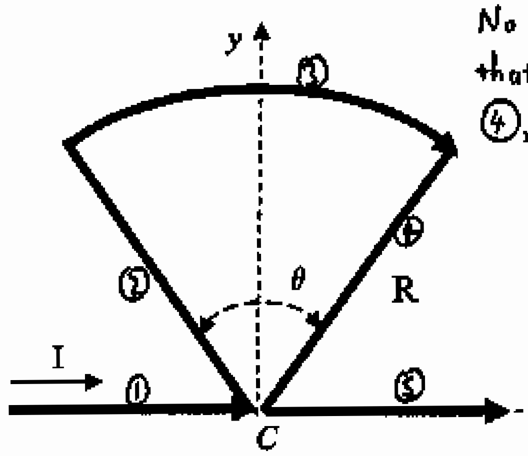
$$= (5\times10^{-4}\ \text{A}\cdot\text{m}^2)(.5\ \text{T}) = \boxed{2.5\times10^{-4}\ \text{J}}$$

Work-kinetic energy theorem

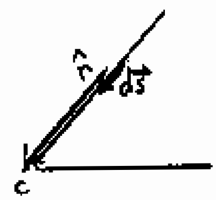
Conservation of energy: $\Delta KE + \Delta PE = 0$

$\Rightarrow \boxed{B}$

4. Calculate the magnitude of the magnetic field at the point C=(0,0) for the current loop shown below. The loop consists of two straight portions and a circular arc of radius R, which subtends an angle $\theta=\pi/2$ at the center of the arc. Ignore the contribution of the current in the short arcs near the origin (0,0).



No contribution to $\vec{B}$-field at C due to arcs that point directly toward or away from C, eg. 4, since $d\vec{s}\times\hat{r}=0$

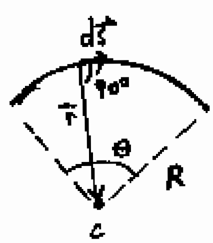


Biot-Savart Law

$$d\vec{B}=\frac{\mu_0 I}{4\pi}\frac{\overbrace{\boxed{d\vec{s}\times\hat{r}}}^{=0}}{r^2}=0$$

(a) $B=\mu_0 I/(8\pi R)$
(b) $B=\mu_0 I/(2\pi R)$
(c) $B=\mu_0 I/(8R)$
(d) $B=\mu_0 I/(2R)$
(e) $B=\mu_0 I/(4R)$

∴ Only arc 3 contributes to $\vec{B}$-field at C



$$dB=\frac{\mu_0 I}{4\pi}\frac{|d\vec{s}\times\hat{r}|}{r^2}=\frac{\mu_0 I}{4\pi}\frac{\overbrace{|d\vec{s}|}^{ds}\,\overbrace{|\hat{r}|}^{1}\,\overbrace{\sin 90^\circ}^{1}}{r^2}$$

$$B=\int_{arc} dB=\int_{arc}\frac{\mu_0 I}{4\pi}\frac{ds}{r^2}=\int_{arc}\frac{\mu_0 I}{4\pi}\frac{ds}{R^2}$$

r = R over arc 3

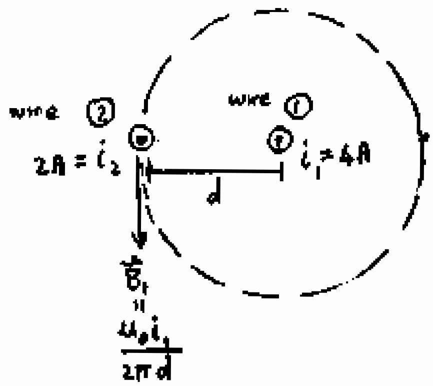
$$\Rightarrow B=\frac{\mu_0 I}{4\pi R^2}\underbrace{\boxed{\int_{arc} ds}}_{\text{arc length of arc 3}\,=\,R\theta}=\frac{\mu_0 I R\theta}{4\pi R^2}=\frac{\mu_0 I\theta}{4\pi R}$$

$$\therefore |\vec{B}| \text{ at } C=\frac{\mu_0 I\theta}{4\pi R}=\frac{\mu_0 I(\frac{\pi}{2})}{4\pi R}=\boxed{\frac{\mu_0 I}{8R}}$$

$\Rightarrow \boxed{C}$

5. Two parallel infinite wires 4 cm apart carry currents of 2A and 4A respectively in the same direction. The force per unit length in N/m of one wire on the other is:

(a) 1×10^{-3} N/m repulsive
(b) 1×10^{-3} N/m attractive
(c) 4×10^{-5} N/m repulsive
(d) 4×10^{-5} N/m attractive
(e) None of these



Let $\vec{\ell}$ be a section of wire ② in the same direction as the current: this section has length $|\vec{\ell}| = \ell$.

The force on this section is

$$\vec{F}_2 = i_2 \vec{\ell} \times \vec{B}_1$$

Since $\vec{\ell}$ points out of the page and the $\vec{B}$-field due to wire ① points down at the position of wire ②, $\vec{\ell} \times \vec{B}$ points to the right, so $\vec{F}_2 = i_2 \vec{\ell} \times \vec{B}_1$ points to the right and hence the force is attractive.

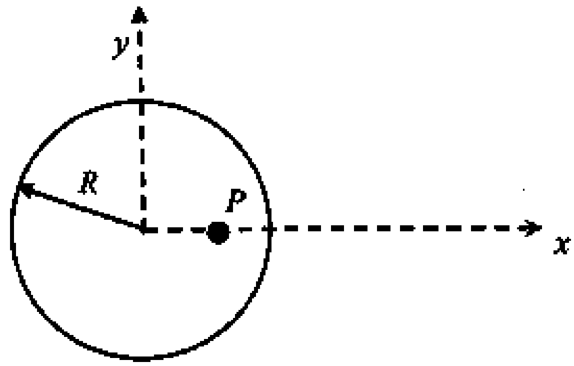
$$|\vec{F}_2| = |i_2 \vec{\ell} \times \vec{B}_1| = i_2 \underbrace{|\vec{\ell} \times \vec{B}_1|}_{= \underbrace{|\vec{\ell}|}_{=\ell} \underbrace{|\vec{B}_1|}_{=\frac{\mu_0 i_1}{2\pi d}} \underbrace{\sin 90^\circ}_{=1}} = \frac{\mu_0 i_1 i_2 \ell}{2\pi d}$$

$$\Rightarrow F_2 = \frac{\mu_0 i_1 i_2 \ell}{2\pi d}$$

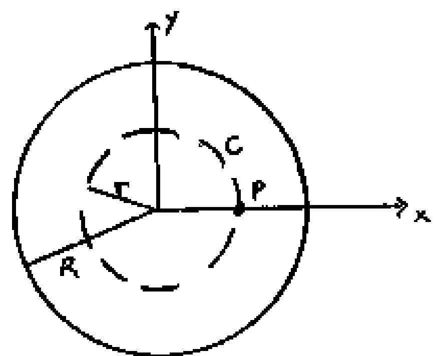
$$\Rightarrow \frac{F_2}{\ell} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{(4\pi \times 10^{-7}\ \frac{T\cdot m}{A})(2\ A)(4\ A)}{2\pi\ (.04\ m)} = \boxed{4\times10^{-5}\ \frac{N}{m},\ \text{attractive}}$$

$$\Rightarrow \boxed{D}$$

6. A long straight wire (radius R= 3mm) carries a constant current distributed uniformly over a cross section perpendicular to the axis of the wire. If the current density is j=100A/m^2 in the $-z$ direction, what is the magnitude and direction of the magnetic field B at the point P located 2 mm from the axis of the wire along the x-axis? (Note that the +z axis points out of the page).



Define an Amperian loop C passing thru. the point at which we want to compute $\vec{B}$; this loop will have a radius r = 2 mm.



(a) 2×10^{-8} T in the +y-direction
(b) 2×10^{-8} T in the –y-direction
(c) 1.3×10^{-7} T in the +y-direction
(d) 1.3×10^{-7} T in the –y-direction
(e) None of these

The current enclosed by the Amperian loop is

$$i_{encl} = j\,\underbrace{(\text{area enclosed by Amperian loop})}_{\pi r^2}$$

$$\Rightarrow i_{encl} = j\pi r^2$$

By Ampere's Law

$$\oint_C \underbrace{\vec{B}\cdot d\vec{s}}_{B\,ds} = \mu_0 i_{encl} = \mu_0 j \pi r^2$$

$$B \boxed{\oint_C ds} = \text{distance around } C = 2\pi r$$

$$\Rightarrow B(r)\cdot 2\pi r = \mu_0 j \pi r^2$$

$$\Rightarrow B(r) = \frac{\mu_0 j \pi r^2}{2\pi r} = \frac{\mu_0 j r}{2} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(100\ \frac{A}{m^2})(2\times10^{-3}\ m)}{2}$$

$$= \boxed{1.3\times10^{-7}\ T}$$

The direction of $\vec{B}$ at P is found by a right-hand rule: point thumb in direction of current (into page), then fingers curl in direction of $\vec{B}$; in particular, the $\vec{B}$-field points down (i.e. in the $-y$ direction) at P.

$$\Rightarrow \boxed{D}$$

7. Two long solenoids (radii 20 mm and 30 mm respectively) carry the came current I, flowing in opposite directions. The smaller solenoid is mounted inside the larger, along a common axis. It is observed that there is a zero magnetic field within the inner solenoid. Therefore the inner solenoid must have X times as many turns per unit length as the outer solenoid, where X is:

(a) 1
(b) 4/9
(c) 2/3
(d) 3/2
(e) 9/4

Let the outer solenoid have: current i_1, $\frac{\text{turns}}{\text{unit length}}$ n_1

and let the inner solenoid have: current i_2, $\frac{\text{turns}}{\text{unit length}}$ n_2

Then inside the inner solenoid, the B-fields due to the two solenoids look like

$$\longrightarrow \vec{B}_1 = \mu_0 i_1 n_1$$
$$\longleftarrow \vec{B}_2 = \mu_0 i_2 n_2$$

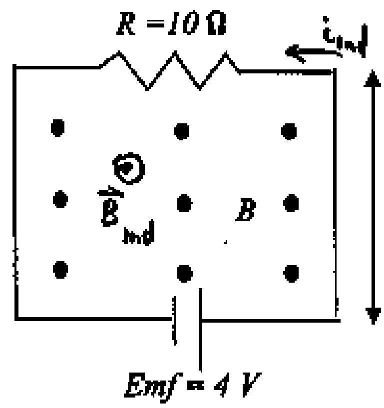
The net magnetic field is zero here when:

$|\vec{B}_1| = |\vec{B}_2|$, i.e. $\mu_0 i_1 n_1 = \mu_0 i_2 n_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{i_2}{i_1} = \boxed{1} \Rightarrow \boxed{A}$

currents are equal by assumption

8. A square loop of wire measuring 12 cm by 12 cm has a battery of emf 4 V and a 10 Ohm resistor. A uniform magnetic field B points out of the page and is decreasing in magnitude at the rate of 150 T/s . The current in the circuit is:



1. Magnetic flux thru one turn is

$$\Phi_{B,1} = \int_{\substack{\text{area encl.}\\ \text{by loop}}} \vec{B}\cdot d\vec{A} = B\underbrace{(\text{area})}_{W^2} = BW^2$$

2. Induced emf

$$\mathcal{E}_{ind} = -\frac{d}{dt}(\underset{\substack{\uparrow\\ N=1}}{N}\Phi_{B,1}) = -\frac{d}{dt}BW^2 = -W^2\frac{dB}{dt}$$

3. Induced current

$$|i_{ind}| = \left|\frac{\text{induced emf } \mathcal{E}_{ind}}{\text{total resistance around loop}}\right| = \left|\frac{-W^2\frac{dB}{dt}}{R}\right| = \frac{W^2}{R}\left|\frac{dB}{dt}\right| = \frac{(.12\text{ m})^2}{10\,\Omega}\left|-150\text{ T/s}\right| = .216\text{ A}$$

(a) 0.62 A, clockwise
(b) 0.22 A, clockwise
(c) 0.62 A, counterclockwise
(d) 0.22 A, counterclockwise
(e) 0 A

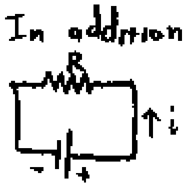
4. Find direction of induced current using Lenz's law:

$|\vec{B}|\downarrow$ with time

$\Rightarrow |\Phi_{B,1}|\downarrow$ with time

To oppose the change in flux, we want to strengthen $\vec{B}$. We can do so by making the induced $\vec{B}$-field point in the same direction as the original $\vec{B}$-field, i.e. out of the page. Right hand rule: point thumb in direction of induced $\vec{B}$-field (out of page), then fingers curl in direction of induced current, i.e. counterclockwise.

5. In addition to the induced current, there is also the current due to the battery

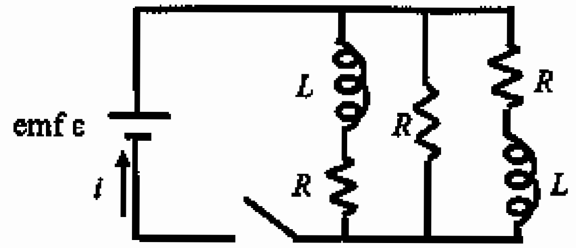


Writing loop equation: $\mathcal{E} - iR = 0 \Rightarrow i = \frac{\mathcal{E}}{R} = \frac{4\text{ V}}{10\,\Omega} = .4\text{ A}$, counterclockwise.

6. total current = .22 A, counterclockwise + .4 A, counterclockwise = $\boxed{.62\text{ A, counterclockwise}}$

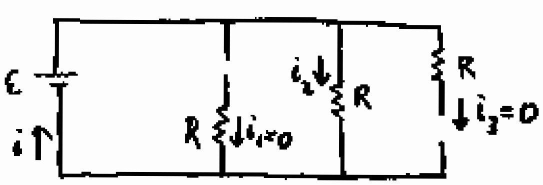
$\Rightarrow \boxed{C}$

9. The figure below shows a circuit that contains three identical resistors with resistance $R = 9\ \Omega$, two identical inductors with inductance $L = 2.0$ mH and an ideal battery with emf $\varepsilon = 18$V. What is the current i through the battery **just after** the switch is closed? What is the current i through the battery a **long time after** the switch has been closed?



	just after	a long time after
(a)	3 A	2 A
(b)	6 A	6 A
(c)	2 A	2 A
(d)	6 A	2 A
(e)	2 A	6 A

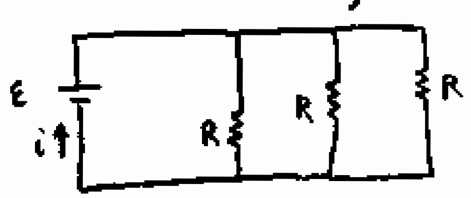
Just after switch is closed, current thru each inductor is zero (because current thru each inductor was zero before the switch was closed, and inductors don't allow the current thru them to change instantaneously). Thus we can replace each inductor with an open circuit:



Since $i_1 = i_3 = 0$, $i = i_1$, where i is the current thru the switch. Writing a loop equation: $\varepsilon - i_2 R = 0$

$\Rightarrow i_2 = \frac{\varepsilon}{R} \Rightarrow i = i_2 = \frac{\varepsilon}{R} = \frac{18\ V}{9\ \Omega} = \boxed{2\ A}$

A long time after switch is closed, current thru each is constant, and therefore (by $\varepsilon_L = -L\frac{di}{dt}$), the voltage across each inductor is zero, so we may as well replace each inductor with a straight wire:



$\Rightarrow$

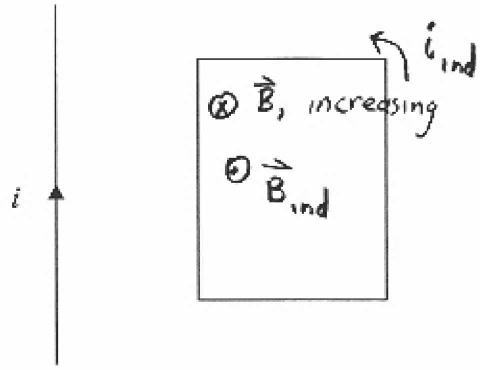
$R_{eq} = \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = \frac{1}{\frac{3}{R}} = \frac{R}{3}$

loop equation: $\varepsilon - i R_{eq} = 0$

$\Rightarrow i = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R/3} = \frac{3\varepsilon}{R} = \frac{3(18\ V)}{9\ \Omega} = \boxed{6\ A}$

$\Rightarrow \boxed{E}$

10. A long straight wire is in the plane of a rectangular conducting loop. The straight wire carries a constant current *i*, as shown below. While the wire is being moved toward the rectangle, the current in the rectangular loop is:

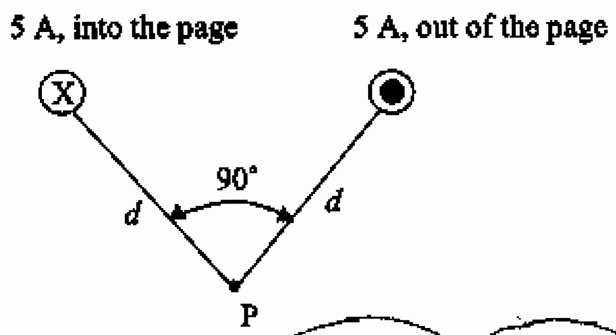


(a) Zero
(b) Clockwise
(c) Counterclockwise
(d) Clockwise on the left side and counterclockwise on the right side
(e) Counterclockwise on the left side and clockwise on the right side

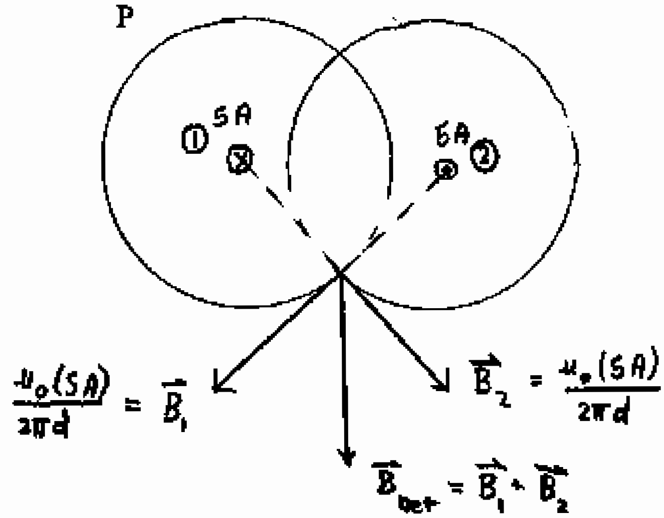
By right hand rule (i.e. point thumb in direction of current, fingers curl in direction of $\vec{B}$) the $\vec{B}$-field points into the page to the right of the wire. Since the wire is being moved toward the loop, the $\vec{B}$-field at the position of the loop is getting stronger and thus the magnitude of the magnetic flux thru the loop, $|\Phi_B|$, is increasing. By Lenz's Law the induced current flows in a direction so as to oppose the change in $|\Phi_B|$. Since $|\Phi_B|$ is increasing with time, we can oppose this change by weakening $\vec{B}$; $\vec{B}$ can be weakened by making the induced $\vec{B}$-field point in the opposite direction to the original $\vec{B}$-field, i.e. out of the page. By another right hand rule (point thumb in direction of induced $\vec{B}$-field, fingers curl in the direction of the induced current) we find that the induced current flows $\boxed{\text{counterclockwise}}$

$\Rightarrow \boxed{C}$

11. Consider two parallel infinite wires, one carrying a current of 5 A out of the page, the other carrying a current of 5 A into the page. The point P is located such that if a line were drawn from P to each wire, then these lines would subtend an angle of 90 degrees, as shown below. Furthermore, the point P is at a distance d from each wire. What is the direction of the magnetic field at the point P?



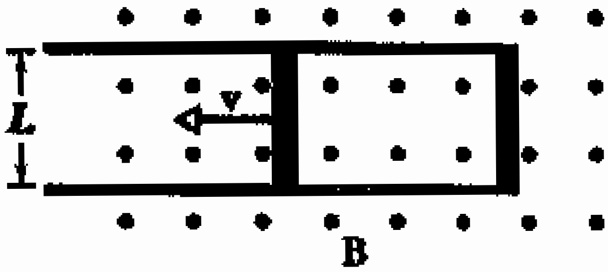
(a) Up
(b) Down
(c) Left
(d) Right
(e) Into the page



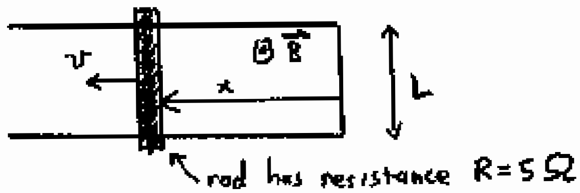
net magnetic field points $\boxed{\text{down}}$.

$\Rightarrow \boxed{B}$

12. A metal rod is forced to move with constant velocity v along two parallel metal rails, connected with a strip of metal at one end, as shown in the figure below. A magnetic field B = 0.5 T points out of the page. If the rails are separated by L = 20 cm and the speed of the rod is v = 10 cm/s, what is the emf generated? If the rod has a resistance of 5 ohms and the rails and connector have negligible resistance, what is the current I in the rod?



(a) emf=0.05 V, I=0.01 A
(b) emf=0.02 V, I=0.004 A
(c) emf=0.01 V, I=0.0 A
(d) emf=0.01 V, I=0.002 A
(e) emf=0 V, I=0 A



Let x = distance of rod from right end.

1. Find magnetic flux thru loop:

$$\Phi_{B,1} = \int_{\substack{\text{area enclosed} \\ \text{by loop}}} \vec{B}\cdot d\vec{A} = |\vec{B}|\underbrace{(\text{area enclosed by loop})}_{Lx} = |\vec{B}|Lx$$

2. Induced emf is

$$\mathcal{E} = -\frac{d}{dt}(N\,\Phi_{B,1}) = -\frac{d}{dt}(|\vec{B}|Lx) = -|\vec{B}|L\boxed{\frac{dx}{dt}} = -|\vec{B}|Lv$$

\# of turns $N = 1$; $\frac{dx}{dt} = v$

$$= -(.5\text{ T})(.20\text{ m})(.1\text{ m/s}) = -.01\text{ V}$$

$$\boxed{|\mathcal{E}| = .01\text{ V}}$$

3. magnitude of induced current is

$$= \frac{|\mathcal{E}|}{\left(\substack{\text{total resistance} \\ \text{around loop}}\right)} = \frac{|\mathcal{E}|}{R} = \frac{.01\text{ V}}{5\ \Omega} = \boxed{.002\text{ A}}$$

→ $\boxed{\text{D}}$