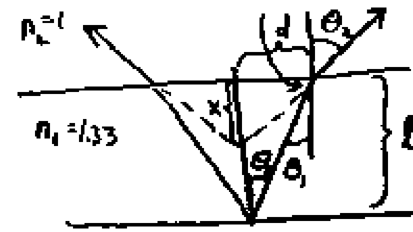
Phys 241 Final
Fall 1996



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$d = \ell \tan\theta_1 \approx \ell \sin\theta_1 = \ell \frac{n_2}{n_1} \sin\theta_2$

$\approx \ell \frac{n_2}{n_1} \tan\theta_2$

$\tan(90-\theta_2) = \frac{x}{d}$

1. The Earth is 1.49×10^{11} meters from the sun. If the solar radiation at the top of the Earth's atmosphere is 1340 W/m², what is the total power output of the sun?

(a) 7×10^{27} W
(b) $2 \times 10^{[illegible]}$ W
(c) $6.62 \times 10^{[illegible]}$ W
$\boxed{\text{(d) } 3.74 \times 10^{26} \text{ W}}$
(e) $2.98 \times 10^{[illegible]}$ W

intensity $I = 1340$ W/m²

P_s = power emitted by source (sun)

r = distance of sun to earth

$I = \frac{P_s}{4\pi r^2} \Rightarrow P_s = I \cdot 4\pi r^2 = (1340\ W/m^2) 4\pi (1.49 \times 10^{11} m)^2$
$= \boxed{3.738 \times 10^{26}\ W}$

2. Green light has a wavelength of 5.4×10^{-7}m. What is the frequency of this EM-wave in air?

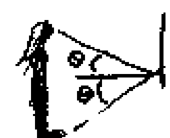
$\boxed{\text{(a) } 5.55 \times 10^{14} \text{ Hz}}$
(b) 6×10^{11} Hz
(c) 9×10^{8} Hz
(d) 3×10^{10} Hz
(e) 1.8×10^{15} Hz

$\lambda = 5.4 \times 10^{-7}$ m

$f = \frac{c}{\lambda} = \frac{3 \times 10^8\ m/s}{5.4 \times 10^{-7}\ m} = \boxed{5.55 \times 10^{14}\ Hz}$

3. A woman 1.70 m tall looks at herself in a full-length mirror (floor-to-ceiling). Where in the mirror must she look to see her feet?

$\boxed{\text{(a) 85 cm from the floor}}$
(b) 50 cm from the floor
(c) 25 cm from the floor
(d) at the bottom of the mirror
(e) 1.5 cm from the floor



Angle of incidence = angle of reflection.
She must look at a point half the distance between her eyes and her feet. $\frac{1.70 m}{2} = \boxed{.85\ m}$

4. A dentist uses a concave mirror (focal length 2 cm) to examine some teeth. If the distance from the object to the mirror is 1 cm, what is the magnification of the tooth?

(a) 6
(b) 1
(c) 4
$\boxed{\text{(d) } 2}$
(e) 1.5

concave $\Rightarrow f > 0$, $f = +2$ cm, $p = 1$ cm

$\frac{1}{i} + \frac{1}{p} = \frac{1}{f} \Rightarrow i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{2 cm} - \frac{1}{1 cm}} = -2$ cm

$m = -\frac{i}{p} = -\frac{(-2 cm)}{1 cm} = \boxed{2}$

5. A 1 meter deep pool of water (n = 1.33) is viewed from overhead. How deep would it appear in cm?

(a) 133
$\boxed{\text{(b) } 75}$
(c) 90
(d) 117
(e) 100

$x = d \tan(90-\theta_2) \approx \left(\ell \frac{n_2}{n_1} \tan\theta_2\right) \tan(90-\theta_2)$

$\tan\theta_2 \tan(90-\theta_2) = 1$ since

$\tan(90-\theta_2) = \frac{\sin(90-\theta_2)}{\cos(90-\theta_2)} = \frac{\cos\theta_2}{\sin\theta_2} = \frac{1}{\tan\theta_2}$

$\Rightarrow x$ = apparent depth of water $= \ell \frac{n_2}{n_1} = (1\ m)\frac{1}{1.33} = \boxed{75.2\ cm}$

6. An object 20 cm high is placed 50 cm in front of a lens whose focal length is 5.0 cm. Where will the image be located (in cm)?

(a) 5.13
$\boxed{\text{(b) } 5.55}$
(c) 5.72
(d) 5.93
(e) 4.55

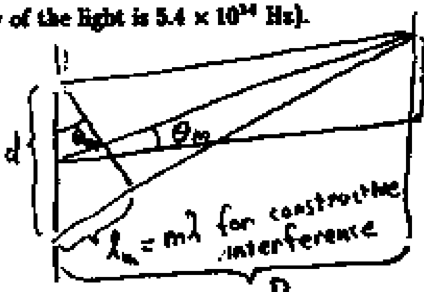
$p = 50$ cm $\quad f = 5$ cm

$\frac{1}{i} + \frac{1}{p} = \frac{1}{f} \Rightarrow i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{5 cm} - \frac{1}{50 cm}} = \boxed{5.55\ cm}$

$i > 0 \Rightarrow$ real image, image on opposite side of lens

7. Light is incident on a double-slit. The fourth bright band has an angular distance of 7° from the central maximum. What is the distance between the slits (in μm)? (Assume the frequency of the light is 5.4×10^{14} Hz).

(a) 27 μm
(b) 21 μm
(c) 24 μm
$\boxed{\text{(d) 18 μm}}$
(e) 14 μm



$d \sin\theta_4 = \ell_4 = 4\lambda$

$\lambda = \frac{c}{f}$

$d = \frac{4\lambda}{\sin\theta_4} = \frac{4c}{f \sin\theta_4}$

$d = \frac{4(3 \times 10^8\ m/s)}{(5.4 \times 10^{14}\ Hz)\sin 7°} = \boxed{18 \times 10^{-6}\ m}$

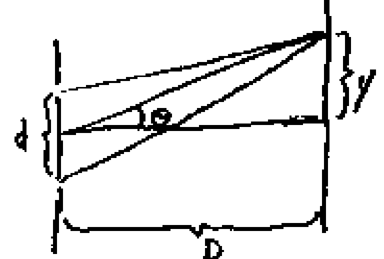
8. Two slits separated by 0.05 mm are illuminated with green light (λ = 540 nm). How many bands of bright lines are there between the central maximum and the 12 cm position? (The distance between the double slits and the screen is 1 m).

(a) 1111
(b) 111
$\boxed{\text{(c) } 11}$
(d) 1
(e) 11111

(refer to above diagram) $\quad d \sin\theta_m = m\lambda$

$\tan\theta_m = \frac{y_m}{D} \approx \sin\theta_m = \frac{m\lambda}{d} \Rightarrow m = \frac{y_m d}{\lambda D} = \frac{(12 \times 10^{-2} m)(.05 \times 10^{-3} m)}{(540 \times 10^{-9} m)(1 m)}$
$= 11.11$

$\Rightarrow \boxed{11 \text{ bands}}$



9. Two slits are illuminated with green light ($\lambda = 540$ nm). The slits are 0.05 mm apart and the distance to the screen is 1.5 m. At what distance (in mm) is the average intensity 50% of the central maximum?

$I = I_o \cdot 4\cos^2\left(\frac{\phi}{2}\right) \Rightarrow \frac{I}{I_o} = \frac{1}{2} = \cos^2\left(\frac{\phi}{2}\right)$

(a) 1
(b) 3
(c) 2
$\boxed{\text{(d) 4}}$
(e) 0.4

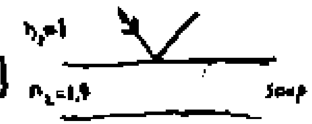
$\Rightarrow \cos^2\left(\frac{\phi}{2}\right) = \frac{1}{2} \Rightarrow \phi =$ [redacted] $\frac{}{90°} \times \frac{\pi \text{ rad}}{180°} =$ [redacted] $\frac{\pi}{2}$ rad

$\phi = \frac{2\pi d}{\lambda}\sin\theta \qquad \tan\theta = \frac{y}{D}$

$y = D\tan\theta \approx D\sin\theta = D\left(\frac{\lambda\phi}{2\pi d}\right) = (1.5\,m)\frac{(540\times10^{-9}\,m)(\frac{\pi}{2}\text{ rad})}{2\pi(.05\times10^{-3}\,m)} = \boxed{4.05}$ [redacted]

10. Monochromatic light ($\lambda = 580$ nm) is incident on a soap bubble ($n = 1.4$) that is 50 nm thick. What is the change of phase of the light reflected from the front surface?

(a) 0°
$\boxed{\text{(b) 180°}}$
(c) $\lambda/2$
(d) $\pi/2$
(e) 55°

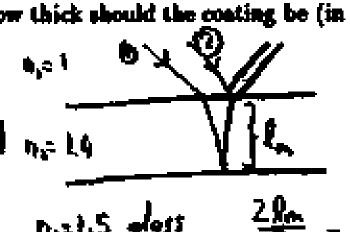


Ray shown is shifted by .5 wavelength (π radians or 180°) when it reflects off medium with higher index of refraction.

Note that answer is not $\lambda/2$ because $\lambda/2$ is a length, whereas the phase, being the argument of a cosine, must have the dimensions of an angle.

11. An optical coating ($n = 1.4$) on a glass lens is designed to minimize reflection at 500 nm. How thick should the coating be (in nm)?

(a) 84
(b) 94
$\boxed{\text{(c) 89}}$
(d) 99
(e) 179



Both rays ① and ② reflect off a medium with a higher index of refraction and so are shifted by .5 wavelength.

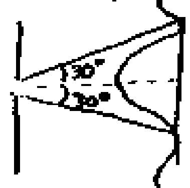
For destructive interference

$\frac{2\ell_m}{\lambda_{n_2}} = m - \frac{1}{2}$, Thickness $\ell_m = \frac{\lambda_{n_2}}{2}(m - \frac{1}{2})$

Try $m=1$: thickness $\ell_1 = \frac{\lambda}{2n_2}(1-\frac{1}{2}) = \frac{500\times10^{-9}\,m}{2(1.4)}(1-\frac{1}{2}) = \boxed{89\text{ nm}}$

12. A narrow slit is illuminated by a sodium-yellow light of wavelength 589 nm. If the central maximum extends to ±30°, how wide is the slit?

(a) 0.5 mm
(b) 2.2×10^{-6} m
(c) 3.3×10^{-6} m
$\boxed{\text{(d) } 1.18\ \mu\text{m}}$
(e) 5.89 μm



1st order minimum at 30°

$a\sin\theta_m = m\lambda$ with $m=1$

width of slit is

$a = \frac{\lambda}{\sin\theta_1} = \frac{589\times10^{-9}\,m}{\sin 30°} = \boxed{1.178\times10^{-6}\,m}$

13. Monochromatic light from a He-Ne laser ($\lambda = 632.8$ nm) is incident on a diffraction grating containing 5000 lines/cm. Determine the angle of the first-order maximum.

$\boxed{\text{(a) 18.4°}}$
(b) 39.2°
(c) 14.6°
(d) 27.7°
(e) 13.9°

d = grating spacing = $\frac{\text{width of grating}}{\#\text{ rulings}} = \frac{W}{N} \qquad \frac{1}{d} = \frac{N}{W} = \frac{\#\text{lines}}{\text{cm}}$

m^{th} maximum ($m = 0, 1, 2, \cdots$) satisfies

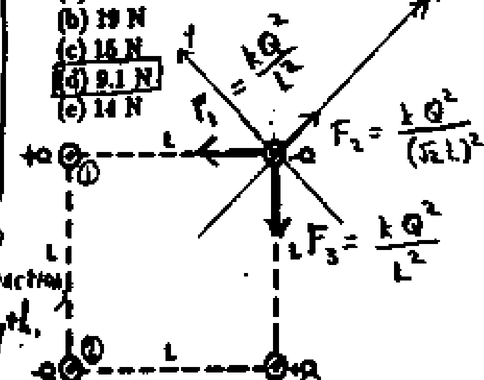
$d\sin\theta_m = m\lambda$

first order maximum $\Rightarrow m=1$

$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\lambda\left(\frac{1}{d}\right)\right) = \sin^{-1}\left((632.8\times10^{-9}\,m)(5000\,\frac{\text{lines}}{\text{cm}} \times \frac{100\text{ cm}}{1\text{ m}})\right) = \boxed{18.4°}$

14. If $Q = 20\mu$C and $L = 60$ cm, what is the magnitude of the electrostatic force on any one of the charges shown?

(a) 25 N
(b) 19 N
(c) 16 N
$\boxed{\text{(d) 9.1 N}}$
(e) 14 N



net force in y direction = 0

$|\vec{F}| = |F_x| = |F_2 - 2F_1\cos 45°|$

$= \left|\frac{kQ^2}{(\sqrt{2}L)^2} - 2\left(\frac{kQ^2}{L}\right)\frac{1}{\sqrt{2}}\right|$

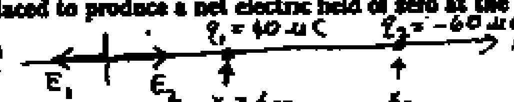
$= \frac{kQ^2}{L^2}\left|\frac{1}{2} - \frac{2}{\sqrt{2}}\right|$

$= \frac{(8.99\times10^9)(20\times10^{-6}\,C)^2}{(.6\,m)^2}\left|\frac{1}{2} - \sqrt{2}\right|$

$= \boxed{9.13\text{ N}}$

15. A 40μC charge is positioned on the x axis at $x = 4.0$ cm. At what x should a -60μC charge be placed to produce a net electric field of zero at the origin?

(a) −5.3 cm
(b) 5.7 cm
$\boxed{\text{(c) 4.9 cm}}$
(d) −6.0 cm
(e) +6.0 cm



$0 = E_x = E_2 - E_1 = \frac{k|q_2|}{x_2^2} - \frac{k|q_1|}{x_1^2} \Rightarrow \frac{|q_2|}{x_2^2} = \frac{|q_1|}{x_1^2}$

$\Rightarrow x_2 = x_1\sqrt{\frac{|q_2|}{|q_1|}} = (4\text{ cm})\sqrt{\frac{|-60\,\mu C|}{|40\,\mu C|}} = \boxed{4.89\text{ cm}}$

16. A charge of uniform volume density (40 nC/m³) fills a cube with 8.0 cm edges. What is the total electric flux through the surface of this cube?

(a) 2.9 $N\cdot m^2/C$
(b) 2.8 $N\cdot m^2/C$
(c) 2.6 $N\cdot m^2/C$
$\boxed{\text{(d) 2.3 } N\cdot m^2/C}$
(e) 1.8 $N\cdot m^2/C$

By Gauss' Law, $\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$
where S is surface of cube, q_{enc} is charge enclosed.
Flux $\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \frac{(40\times10^{-9}\,C/m^3)(.08\,m)^3}{8.85\times10^{-12}\,\frac{C^2}{Nm^2}}$
$= \boxed{2.31\ \frac{Nm^2}{C}}$

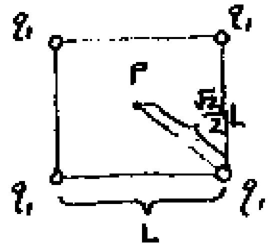
17. A long non-conducting cylinder (radius = 12 cm) has a charge of uniform density (5.0 nC/m³) distributed throughout its volume. Determine the magnitude of the electric field 15 cm from the axis of cylinder. Let R = radius of cylinder. Take the

(a) 20 N/C
$\boxed{\text{(b) 27 N/C}}$
(c) 16 N/C
(d) 12 N/C
(e) 54 N/C

Gaussian surface S to be a cylinder of length L and radius r = 15 cm. By Gauss' Law, $\oint_S \vec{E}\cdot d\vec{A} = \frac{\text{charge in } S}{\epsilon_0}$
or $E\cdot 2\pi r L = \frac{\rho \pi R^2 L}{\epsilon_0} \Rightarrow E = \frac{\rho \pi R^2}{2\pi r \epsilon_0} = \frac{\rho R^2}{2 r \epsilon_0}$
$E = \frac{(5\times10^{-9}\,C/m^3)(.12\,m)^2}{2(.15\,m)(8.85\times10^{-12})} = \boxed{27.11\ N/C}$

18. Identical 2.0µC charges are located on the vertices of a square with sides that are 2.0 m in length. Determine the electric potential (relative to zero at infinity) at the center of the square.

(a) 38 kV
$\boxed{\text{(b) 51 kV}}$
(c) 76 kV
(d) 64 kV
(e) 13 kV



$V(P) = 4\,\frac{kq_1}{\frac{\sqrt{2}}{2}L}$
$= 4\,\frac{(8.99\times10^9)(2\times10^{-6}\,C)}{\frac{\sqrt{2}}{2}(2\,m)}$
$= \boxed{50.8\times10^3\ V}$

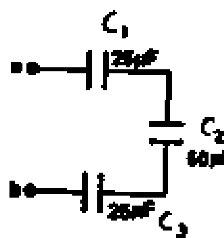
19. In the figure, if $V_a - V_b$ = 22V, how much energy is stored in the 50µF capacitor?

(a) 0.78 mJ
(b) 0.58 mJ
(c) 0.68 mJ
$\boxed{\text{(d) 0.48 mJ}}$
(e) 0.22 mJ

a, b — C_{123}

$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{25} + \frac{1}{50} + \frac{1}{25} = \frac{5}{50}$
$\Rightarrow C_{123} = 10\ \mu F$

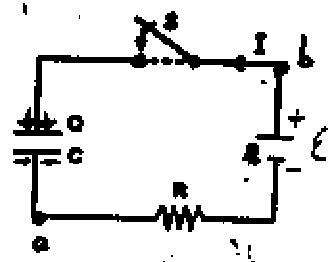
Charge on $C_{123} = q = C_{123}(V_a - V_b) = (10\times10^{-6}F)(22\ V)$
$= 2.2\times10^{-4}\ C$



Capacitors in series have same charge on each, so charge on C_2 is q.
energy stored in $C_2 = \frac{1}{2}\frac{q^2}{C_2} = \frac{1}{2}\frac{(2.2\times10^{-4}\,C)^2}{50\times10^{-6}F}$
$= \boxed{.48\times10^{-3}\ J}$

20. In the figure, at t = 0 the switch S is closed with the capacitor uncharged. If C = 50µF $\mathcal{E}$ = 20 V, and R = 4.0 kΩ, what is the charge on the capacitor when I = 2.0 mA?

(a) 360 µC
(b) 480 µC
(c) 240 µC
$\boxed{\text{(d) 600 µC}}$
(e) 400 µC



$V_a - IR + \varepsilon = V_b \Rightarrow V_b - V_a = \varepsilon - IR$
Voltage across capacitor is $V_b - V_a$
$\Rightarrow q = C(V_b - V_a) = C(\varepsilon - IR)$
$= (50\times10^{-6}F)(20\,V - (2\times10^{-3}A)(4\times10^3\Omega))$
$= \boxed{600\ \mu C}$

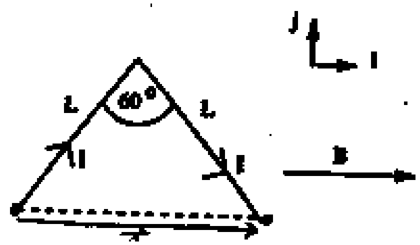
21. A 2.0 C charge moves with a velocity of $(2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k})$ m/s and experiences a magnetic force of $(4.0\hat{i} - 20\hat{j} + 12\hat{k})$ N. The x component of the magnetic field is equal to zero. Determine the y component of the magnetic field.

(a) − 3.0 T
$\boxed{\text{(b) + 3.0 T}}$
(c) + 5.0 T
(d) − 5.0 T
(e) + 6.0 T

$\vec{v} = (2\hat{i} + 4\hat{j} + 6\hat{k})$ m/s $\quad \vec{F} = (4\hat{i} - 20\hat{j} + 12\hat{k}\ N)$
$\vec{B} = B_y\hat{j} + B_z\hat{k} \quad \vec{v}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 0 & B_y & B_z \end{vmatrix} = \hat{j}(-2B_z) + \hat{k}(2B_y)$
from $\vec{F} = q\,\vec{v}\times\vec{B}$
$F_z = q(\vec{v}\times\vec{B})_z = q(2B_y) \Rightarrow B_y = \frac{F_z}{2q} = \frac{12}{2(2)} = \boxed{3\ T}$

22. A straight wire is bent into the shape shown. Determine the net magnetic force on the wire.

$\boxed{\text{(a) Zero}}$
(b) IBL in the +z direction
(c) IBL in the −z direction
(d) 1.7 IBL in the +z direction
(e) 1.4 IBL in the −z direction



draw vector $\vec{\ell}$ from initial point to final point on path.
$\vec{F} = I\underbrace{\vec{\ell}\times\vec{B}}_{0} = 0$ since $\vec{\ell}$ is parallel to $\vec{B}$

23. An electron follows a circular path (radius = 15 cm) in a uniform magnetic field $B = 3 \times 10^{-4}$ T. What is the period of this motion?

$\boxed{\text{(a) 0.12 ms}}$
(b) 1.2 ms
(c) 0.18 μs
(d) 1.8 ms
(e) 1.8 μs

centripetal acceleration $a = \frac{v^2}{r}$ $\quad ma = F_m = qvB$

$\Rightarrow \frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv^2}{qvB} = \frac{mv}{qB}$

period T = time for 1 orbit = $\frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right)$

$= \frac{2\pi m}{qB} = \frac{2\pi(9.11\times10^{-31}\text{ kg})}{(1.6\times10^{-19}\text{ C})(3\times10^{-4}\text{ T})} = \boxed{.119\times10^{-6}\text{ s}}$

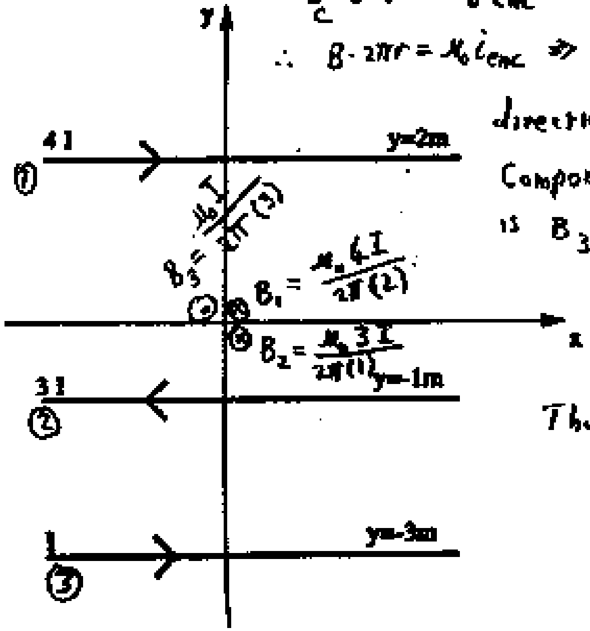
24. Three infinitely long wires parallel to the x axis carry currents as shown. If $I = 20$ A, what is the magnitude of the magnetic field at the origin?

(a) 37 μT
(b) 28 μT
$\boxed{\text{(c) 19 μT}}$
(d) 47 μT
(e) 56 μT

Magnetic field a distance r from wire can be found using Amperes law. Consider an Amperian loop C of radius r that encircles the wire.

$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$ where i_{enc} is the current enclosed

$\therefore B\cdot 2\pi r = \mu_0 i_{enc} \Rightarrow B = \frac{\mu_0 i_{enc}}{2\pi r}$



direction of $\vec{B}$ given by right hand rule.

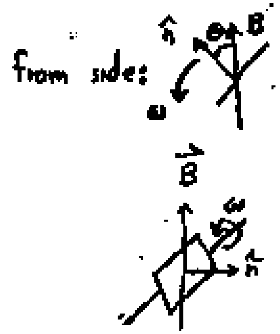
Component of $\vec{B}$ out of page at origin is $B_3 - B_1 - B_2 = \frac{\mu_0 I}{2\pi(3)} - \frac{\mu_0 4I}{2\pi(2)} - \frac{\mu_0 3I}{2\pi(1)} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{3} - \frac{4}{2} - 3\right) = -\frac{14}{3}\frac{\mu_0 I}{2\pi} = -\frac{7\mu_0 I}{3\pi} = -\frac{7(4\pi\times10^{-7})(20\text{ A})}{3\pi}$

$= -18.6\times10^{-6}$ T

Thus $|\vec{B}| = \boxed{18.6\ \mu\text{T}}$

25. A square loop (length along one side = 20 cm) rotates in a constant magnetic field which has a magnitude of 2.0 T. At an instant when the angle between the field and the normal to the plane of the loop is equal to 20° and increasing at the rate of 10°/s, what is the magnitude of the induced emf in the loop?

(a) 13 mV
(b) 0.27 V
$\boxed{\text{(c) 4.8 mV}}$
(d) 14 mV
(e) 2.2 mV



dA = element of area
$\hat{n}$ = normal to plane of loop
$d\vec{A} = \hat{n}\, dA$

magnetic flux $\Phi_B = \int_{loop} \vec{B}\cdot d\vec{A} = \int_{loop} \vec{B}\cdot\hat{n}\, dA$

or $\Phi_B = \int_{loop} |\vec{B}|\underbrace{|\hat{n}|}_{1}\cos\theta\, dA = \int_{loop} B\cos\theta\, dA$

But $\theta = \omega t$

$\therefore \Phi_B = \int_{loop} B\cos(\omega t)\, dA = B\cos(\omega t)\int_{loop} dA$ area of loop $= a^2$ where a is length of side,

$\Phi_B = B a^2 \cos(\omega t)$,

where angular freq. $\omega = 10°/s \times \frac{\pi \text{ rad}}{180°} = .174 \frac{\text{rad}}{s}$

Assume # of turns $N = 1$.

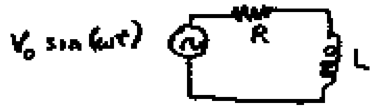
$\mathcal{E} = -N\frac{d\Phi}{dt} = -Ba^2(-\omega\sin(\omega t)) = Ba^2\omega\sin(\omega t)\Big|_{\omega t = \theta = 20°}$

$= (2\text{ T})(.2\text{ m})^2(.174 \frac{\text{rad}}{s})\sin(20°) = \boxed{4.76\times10^{-3}\text{ mV}}$

26. A 2 mH inductor in series with a 2 kΩ resistor is connected to a 60 Hz ac source. Calculate the impedance of this circuit.

(a) 500 ohms
(b) 1000 ohms
$\boxed{\text{(c) 2000 ohms}}$
(d) 3000 ohms
(e) 2400 ohms

$\omega = 2\pi f = 2\pi(60\text{ Hz}) = 377 \frac{\text{rad}}{s}$



impedance $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$

$|Z| = \sqrt{(2\times10^3\ \Omega)^2 + (377 \frac{\text{rad}}{s})^2(2\times10^{-3}\text{ H})^2} = \boxed{2000.0\ \Omega}$