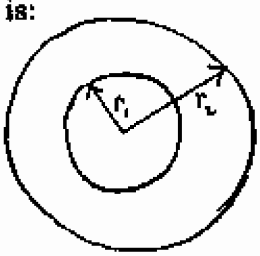
Physics 241
Final May 5, 2001

1. (10 points) Identical point charges are located at two vertices of an equilateral triangle. A third charge is placed so the electric field at the third vertex is zero. The third charge must:

[(A)( )] be on the perpendicular bisector of the line joining the first two charges
(B)( ) be on the line joining the first two charges
(C)( ) be identical to the first two charges
(D)( ) have the same magnitude as the first two charges but may have a different sign
(E)( ) be at the center of the triangle

2. (10 points) A spherical shell has an inner radius of 3.7 cm and an outer radius of 4.5 cm. If charge is distributed uniformly throughout the shell with a volume density of 6.1×10^{-4} C/m^3 the total charge is:

[(A)( ) 1.0×10^{-7} C]
(B)( ) 1.3×10^{-7} C
(C)( ) 2.0×10^{-7} C
(D)( ) 2.3×10^{-7} C
(E)( ) 4.0×10^{-7} C



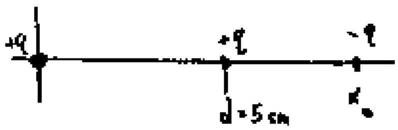
$Q = \rho\left(\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3\right)$
$= \frac{4\pi\rho}{3}(r_2^3 - r_1^3)$
$= \frac{4\pi}{3}(6.1\times10^{-4}\ \frac{C}{m^3})(.045^3 - .037^3)$
$= 1.03\times10^{-7}$ C

3. (10 points) Two identical charges q are placed on the x axis, one at the origin and the other at $x = 5$ cm. A third charge $-q$ is placed on the x axis so the potential energy of the three charge system is the same as the potential energy at infinite separation. Its x coordinate is:

$U = k\left(\frac{q^2}{d} + \frac{q(-q)}{x_0 - d} + \frac{q(-q)}{x_0}\right) = 0 \Rightarrow \frac{1}{d} = \frac{1}{x_0-d} + \frac{1}{x_0} = \frac{x_0 + x_0 - d}{x_0(x_0-d)}$

$\Rightarrow x_0(x_0-d) = d(2x_0-d) \Rightarrow x_0^2 - x_0 d = 2dx_0 - d^2 \Rightarrow x_0^2 - 3dx_0 + d^2 = 0$

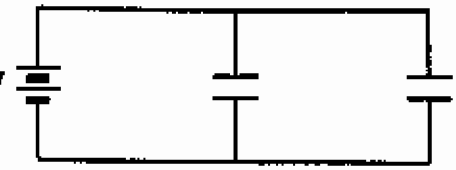
[(A)( ) 13 cm]
(B)( ) 2.5 cm
(C)( ) 7.5 cm
(D)( ) 10 cm
(E)( ) −5 cm



$\Rightarrow x_0 = \frac{3d \pm \sqrt{9d^2 - 4d^2}}{2}$
$= d\frac{(3\pm\sqrt{5})}{2}$
$= (5\text{ cm})\frac{(3\pm\sqrt{5})}{2} = 13.09$ cm, 1.91 cm

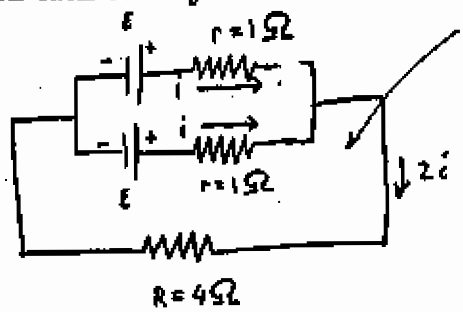
4. (10 points) The two capacitors shown each have a capacitance of 1μF. Their total stored energy is: $U = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 = (1\times10^{-6}\text{ F})(200\text{ V})^2 = .04$ J

(A)( ) 0.01 J
(B)( ) 0.02 J
[(C)( ) 0.04 J]
(D)( ) 0.06 J
(E)( ) none of these



5. (10 points) Two identical batteries, each with an emf of 18 V and an internal resistance of 1 Ω, are wired in parallel by connecting their positive terminals together and connecting their negative terminals together. The combination is then wired across a 4 Ω resistor. The current in each battery is:

(A)( ) 1.0 A
(B)( ) 2.0 A
(C)( ) 4.0 A
(D)( ) 3.6 A
(E)( ) 7.2 A



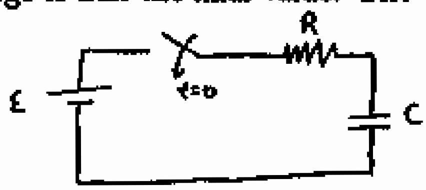
loop eqn: $\mathcal{E} - ir - (2i)R = 0$

$\Rightarrow \mathcal{E} = i(r + 2R)$

$\Rightarrow i = \frac{\mathcal{E}}{r+2R} = \frac{18\text{ V}}{1+2(4)} = \boxed{2\text{ A}}$

6. (10 points) A certain capacitor, in series with a 720 Ω resistor, is being charged. At the end of 10 ms its charge is half the final value. The capacitance is about:

(A)( ) 9.6 μF
(B)( ) 14 μF
(C)( ) 20 μF
(D)( ) 7.2 μF
(E)( ) 10 F



Equation for charging a capacitor:

$q(t) = q_f(1 - e^{-t/\tau_c})$

where $q_f = C\mathcal{E}$, $\tau_c = RC$

Let $t_1 = 10$ ms.

$q(t_1) = \frac{1}{2} q_f \Rightarrow \frac{1}{2} q_f = q(t_1) = q_f(1 - e^{-t_1/\tau_c})$

$\Rightarrow \frac{1}{2} = 1 - e^{-t_1/\tau_c} \Rightarrow e^{-t_1/\tau_c} = \frac{1}{2} \Rightarrow -t_1/\tau_c = \ln(\frac{1}{2}) = -\ln 2 \Rightarrow \tau_c = \frac{t_1}{\ln 2}$, $\tau_c = RC$

$\Rightarrow C = \frac{t_1}{R \ln 2} = \frac{10 \times 10^{-3}\text{ s}}{(720\,\Omega)\ln 2} = 2.0 \times 10^{-5}\text{ F} = \boxed{20\,\mu\text{F}}$

7. (10 points) A proton (charge e), traveling perpendicular to a magnetic field, experiences the same force as an α particle (charge $2e$) which is also traveling perpendicular to the same field. The ratio of their speeds, $v_{\text{proton}}/v_\alpha$, is:

(A)( ) 0.5
(B)( ) 1
(C)( ) 2
(D)( ) 4
(E)( ) 8

$|\vec{F}_{\text{proton}}| = e|\vec{v}_{\text{proton}} \times \vec{B}| = e\, v_{\text{proton}} B$

$\|$

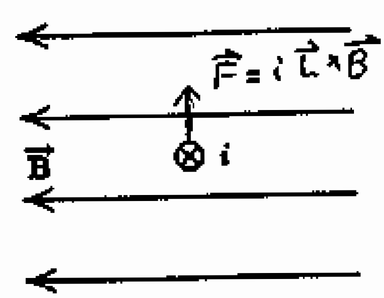
$|\vec{F}_\alpha| = (2e)|\vec{v}_\alpha \times \vec{B}| = 2e\, v_\alpha B$

$\Rightarrow e v_{\text{proton}} = 2e\, v_\alpha$

$\Rightarrow \frac{v_{\text{proton}}}{v_\alpha} = 2$

8. (10 points) The figure shows a uniform magnetic field $\vec{B}$ directed to the left and a wire carrying a current into the page. The magnetic force acting on the wire is:

(A)( ) toward the top of the page
(B)( ) toward the bottom of the page
(C)( ) toward the left
(D)( ) toward the right
(E)( ) zero



9. (10 points) Two long parallel straight wires carry equal currents in opposite directions. At a point midway between the wires, the magnetic field they produce is:

(A)( ) zero
(B)( ) non-zero and along a line connecting the wires
(C)( ) non-zero and parallel to the wires
(D)( ) non-zero and perpendicular to the plane of the two wires in which the two wires lie.
(E)( ) none of the above

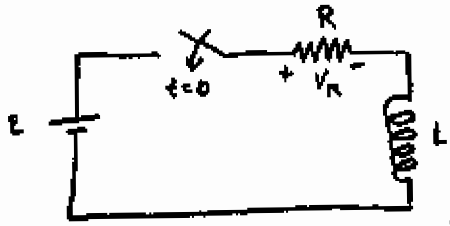
10. (10 points) A solenoid is 3.0 cm long and has a radius of 0.50 cm. It is wrapped with 500 turns of wire carrying a current of 2.0 A. The magnetic field in tesla at the center of the solenoid is:

$$B_{inside} = \mu_0 i n = \mu_0 i \frac{N}{\ell} = (4\pi\times10^{-7}\ \tfrac{T\cdot m}{A})(2\ A)\frac{(500\ \text{turns})}{3\times10^{-2}\ m} = 4.19\times10^{-2}\ T$$

(A)( ) 9.9×10^{-8}
(B)( ) 1.3×10^{-3}
(C)( ) 4.2×10^{-2}
(D)( ) 16
(E)( ) none of these

11. (10 points) An 8.0 mH inductor and a 2.0 Ω resistor are wired in series to a 20 V ideal battery. A switch in the circuit is closed at time 0. After a long time the potential difference across the resistor and the emf of the inductor are:

(A)( ) 20 V, 20 V
(B)( ) 0, 20 V
(C)( ) 10 V, 10 V
(D)( ) 20 V, 0
(E)( ) 0, 0



After a long time, current i is constant ⇒

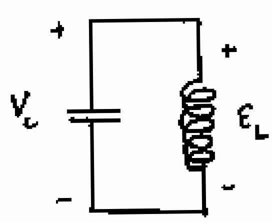
$$\mathcal{E}_L = L\frac{di}{dt} = 0$$

loop equation: $\mathcal{E} - V_R - \mathcal{E}_L = 0$

$$\Rightarrow V_R = \mathcal{E} = 20\ V$$

12. (10 points) A capacitor in an LC oscillator has a maximum potential difference of 15 V and a maximum energy of 360 μJ. At a certain instant the energy in the capacitor is 40 μJ. At that instant what is the emf induced in the inductor?

(A)( ) zero
(B)( ) 5 V
(C)( ) 10 V
(D)( ) 15 V
(E)( ) 20 V



$$E_{max} = \tfrac{1}{2}CV_{c,max}^2 \quad (1)$$

$$E = \tfrac{1}{2}CV_c^2 \quad (2)$$

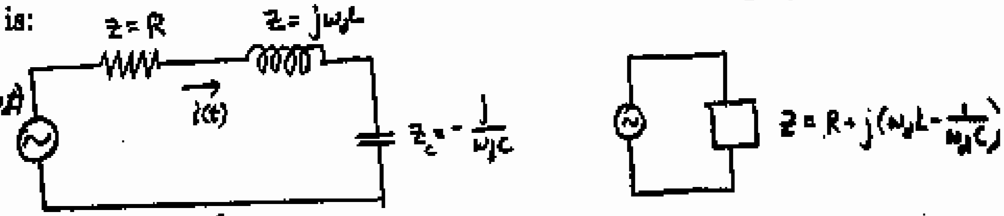
Divide (2) by (1): $\frac{E}{E_{max}} = \frac{\frac{1}{2}CV_c^2}{\frac{1}{2}CV_{c,max}^2} = \frac{V_c^2}{V_{c,max}^2}$

$$\Rightarrow V_c^2 = V_{c,max}^2 \frac{E}{E_{max}} \Rightarrow V_c = V_{c,max}\sqrt{\frac{E}{E_{max}}}$$

But by Kirchoff's voltage law, $\mathcal{E}_L = V_c$

$$\Rightarrow \mathcal{E}_L = V_c = V_{c,max}\sqrt{\frac{E}{E_{max}}} = (15\ V)\sqrt{\frac{40\ \mu J}{360\ \mu J}} = (15\ V)\sqrt{\frac{1}{9}} = 5\ V$$

13. (10 points) An ac generator producing 10 V (rms) at 200 rad/s is connected in series with a 50 Ω resistor, a 400 mH inductor, and a 200 μF capacitor. The rms voltage (in volts) across the capacitor is:



(A)( ) 2.5
(B)( ) 3.4
(C)( ) 6.7
(D)( ) 10.0
(E)( ) 10.8

$$I_{rms} = \frac{\mathcal{E}_{rms}}{|Z|}$$

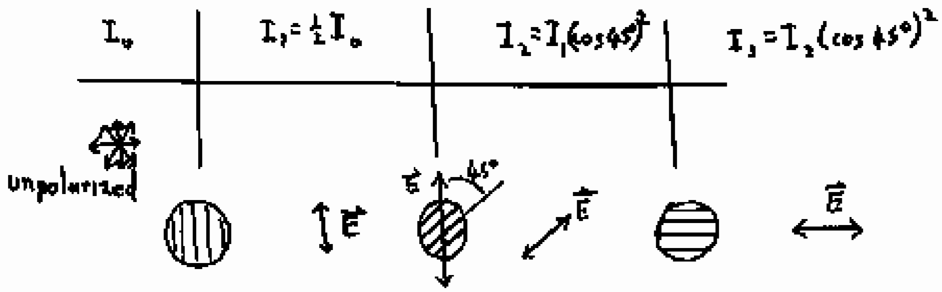
$$V_{c,rms} = I_{rms}|Z_c| = \frac{\mathcal{E}_{rms}}{|Z|}|Z_c| = \mathcal{E}_{rms}\frac{\left|-\frac{1}{\omega_d C}\right|}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

$$= (10\ V)\frac{\frac{1}{(200\ rad/s)(200\times10^{-6}\ F)}}{\sqrt{(50\ \Omega)^2 + \left((200\ rad/s)(400\times10^{-3}\ H) - \frac{1}{(200\ rad/s)(200\times10^{-6}\ F)}\right)^2}}$$

$$= \boxed{3.36\ V}$$

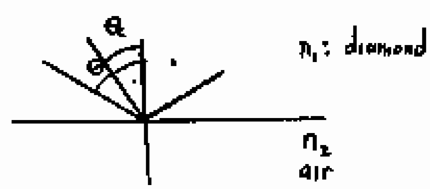
14. (10 points) In a stack of three polarizing sheets the first and third are crossed while the middle one has its axis at 45° to the axis of the other two. The fraction of the intensity of an incident unpolarized beam of light that is transmitted by the stack is:

(A)( ) 1/2
(B)( ) 1/3
(C)( ) 1/4
(D)( ) 1/8
(E)( ) 0



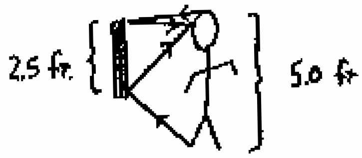
$$I_3 = I_2(\cos 45°)^2 = I_1(\cos 45°)^2(\cos 45°)^2 = I_0\tfrac{1}{2}(\cos 45°)^2(\cos 45°)^2 \Rightarrow \frac{I_3}{I_0} = \tfrac{1}{2}(\cos 45°)^4 = \tfrac{1}{2}\left(\tfrac{1}{\sqrt{2}}\right)^4 = \tfrac{1}{2}\left(\tfrac{1}{2}\right)^2 = \tfrac{1}{8}$$

15. (10 points) The critical angle for total internal reflection at a diamond air interface is 25°. Suppose light is incident at an angle of θ with the normal. Total internal reflection will occur if the incident medium is:



(A)( ) air and θ = 25°
(B)( ) diamond and θ < 25°
(C)( ) air and θ > 25°
(D)( ) diamond and θ > 25°
(E)( ) air and θ < 25°

16. (10 points) A 5.0 ft woman wishes to see a full length image of herself in a plane mirror. The minimum length mirror required is:



(A)( ) 5 ft
(B)( ) 10 ft
(C)( ) 2.5 ft
(D)( ) 3.54 ft
(E)( ) no answer: the farther away she stands the smaller the required mirror length

17. (10 points) A concave spherical mirror has a focal length of 12 cm. If an object is placed 18 cm in front of it the image position is:

$f = 12$ cm $\quad p = 18$ cm

$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{p}$

$\Rightarrow i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{12} - \frac{1}{18}} = 36$ cm

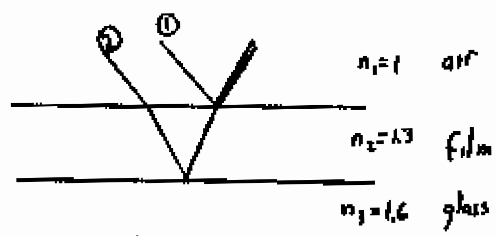
$i > 0 \Rightarrow$ image is real (occurs in front of mirror).

(A)( ) 7.2 cm behind the mirror
(B)( ) 7.2 cm in front of the mirror
(C)( ) 36 cm behind the mirror
(D)( ) 36 cm in front of the mirror
(E)( ) at infinity

18. (10 points) In a Young's double-slit experiment the center of a bright fringe occurs wherever waves from the slits differ in the distance they travel by a multiple of:

(A)( ) a fourth of a wavelength
(B)( ) a half a wavelength
(C)( ) a wavelength
(D)( ) three-fourths of a wavelength
(E)( ) none of the above

19. (10 points) A glass ($n = 1.6$) lens is coated with a thin film ($n = 1.3$) to reduce reflection of certain incident light. If λ is the wavelength of the light in the film, the least film thickness is:



(A)( ) less than $\lambda/4$
(B)( ) $\lambda/4$
(C)( ) $\lambda/2$
(D)( ) λ
(E)( ) more than λ

Both ① and ② reflect off a medium with a higher index of ref. $\Rightarrow$ Both shifted by .5 wavelength.

$\therefore$ Want additional distance traveled by ray ② to be $\lambda/2 \Rightarrow$ least film thickness $= \frac{\lambda}{4}$

20. (10 points) Monochromatic light is normally incident on a grating that is 1 cm wide and has 10,000 slits. The first order line is deviated at a 30° angle. What is the wavelength, in nm, of the incident light?

grating spacing $d \approx \frac{\text{width}}{N} = \frac{1 \text{ cm}}{10{,}000 \text{ rulings}}$

$m = 1 \Rightarrow d \sin\theta_1 = \lambda$ (condition for first maximum)

$\Rightarrow \lambda = d \sin\theta_1 = \frac{1 \text{ cm}}{10{,}000} \sin 30° = 5 \times 10^{-5} \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{10^9 \text{ nm}}{1 \text{ m}} = 500$

(A)( ) 300
(B)( ) 400
(C)( ) 500
(D)( ) 600
(E)( ) 1000