MIXTURES OF CHARGED BOSONS CONFINED IN HARMONIC TRAPS AND BOSE–EINSTEIN CONDENSATION MECHANISM FOR LOW-ENERGY NUCLEAR REACTIONS AND TRANSMUTATION PROCESSES IN CONDENSED MATTERS

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A mixture of two different species of positively charged bosons in harmonic traps is considered in the mean-field approximation. It is shown that depending on the ratio of parameters, the two components may coexist in same regions of space, in spite of the Coulomb repulsion between the two species. Application of this result is discussed for the generalization of the Bose–Einstein condensation mechanism for low-energy nuclear reaction (LENR) and transmutation processes in condensed matters. For the case of deutron–lithium (d + Li) LENR, the result indicates that (d + ^{6}Li) reactions may dominate over (d + d) reactions in LENR experiments.

1. Introduction

In order to understand and explain the anomalous nuclear low-energy reaction phenomena,[1] Bose–Einstein condensation (BEC) of integer-spin nuclei was suggested as a possible mechanism for ultra low-energy nuclear reaction in 1998.[2] Recently, theoretical studies of BEC mechanism have been carried out by approximately solving a many-body Schrödinger equation for a system of N identical charged integer-spin nuclei ("Bose" nuclei) confined in ion traps.[3–7] The solution is used to obtain theoretical formulae for estimating the probabilities and rates of nuclear fusion for N identical Bose nuclei confined in an ion trap or an atomic cluster. In this paper, we generalize our previous one specie BEC mechanism to the two-species case in order to apply our formulation to the LENR and transmutation processes in condensed matters.[8,9]

2. One Specie Case

For the BEC mechanism, the total nuclear d–d fusion rate $R^{(1)}$ per unit volume per unit time is given by[3–7]

$$R^{(1)} = n_{\mathrm{B}}\sqrt{\frac{3}{4\pi}}\Omega B\alpha\left(\frac{\hbar c}{m}\right)Nn_{\mathrm{B}}, \tag{1}$$

where B is given by $B = 3Am/8\pi\alpha\hbar c$, n_{B} is a trap/cluster number density (number of traps/clusters per unit volume) as defined as, $n_{\mathrm{B}} = N_{\mathrm{B}}/N, N_{\mathrm{B}}$ is the total number of Bose nuclei in traps/clusters per unit volume, and N is the average number of Bose nuclei in a trap/cluster. n_{B} is an average number of Bose nuclei per trap/cluster, $n_{\mathrm{B}} = N/\langle r\rangle^3$, where $\langle r\rangle$ is the average size of traps/atomic clusters. A is given by $A = 2Sr_{\mathrm{B}}/\pi\hbar$, where $r_{\mathrm{B}} = \hbar^2/2\mu e^2$, $\mu = m/2$, S is the S-factor for the nuclear fusion reaction between two deutrons (for D(d,p)T and D(d,n)^{3}He reactions, $S \approx 55\,\mathrm{keV}$-barn), and Ω is the probability of the BEC ground-state occupation.

In terms of S-factor, Eq. (1) can be rewritten as

$$R^{(1)} = n_{\mathrm{B}}\Omega K\left(\frac{S}{\mu}\right)Nn_{\mathrm{B}}, \tag{2}$$

where

$$K = \frac{3\sqrt{3}}{8\pi^2\sqrt{4\pi}}\frac{1}{\alpha c}.$$

We note a very important fact that $R^{(1)}$ does not depend on the Gamov factor in contrast to the conventional theory for nuclei fusion in free space. This is consistent with conjecture noted by Dirac[10] that each interacting neutral boson behaves as an independent particle in a common average background for the large N case. Furthermore, the reaction rate $R^{(1)}$ is proportional to Ω which is expected to increase as the operating temperature decreases. The only unknown parameter in Eqs. (1) and (2) is the probability of the BEC ground state occupation, Ω.

Our predictions imply that nuclear fusion may be achievable at lower temperatures.

3. Two Species Case

We consider a mixture of two different species of positive charged bosons, labeled 1 and 2 with N_1 and N_2 particles, respectively. We denote charges and masses as $Z_1 \geq 0$, $Z_2 \geq 0$ and m_1, m_2, respectively. We assume that trapping potentials V_i are isotropic and harmonic

$$V_i(\vec{r}) = m_i\omega_i^2\frac{r^2}{2}.$$

The mean-field energy functional for the two-component system is given by generalization of the one-component case[5]

$$E = \sum_{i=1}^{2} E_i + E_{\mathrm{int}}, \tag{3}$$

where

$$E_i = \int \mathrm{d}\vec{r}\frac{\hbar^2}{2m_i}|\nabla\psi_i|^2,$$

$$E_{\mathrm{int}} = \frac{e^2}{2}\int \mathrm{d}\vec{x}\,\mathrm{d}\vec{y}\frac{(Z_1n_1(\vec{x}) + Z_2n_2(\vec{x}))(Z_1n_1(\vec{y}) + Z_2n_2(\vec{y}))}{|\vec{x} - \vec{y}|},$$

and n_i denotes density of species i, $n_i = |\psi_i|^2$,

$$\int \mathrm{d}\vec{r}\, n_i(\vec{r}) = N_i. \tag{4}$$

In Eq. (3), we have neglected effects of order $1/N_i$.

The minimization of the functional, Eq. (3), with subsidiary conditions, Eq. (4), leads to the following time-independent mean-field equations:

$$-\frac{\hbar^2}{2m_i}\nabla^2\psi_i(\vec{r}) + (V_i + W_i)\psi_i(\vec{r}) = \mu_i\psi_i(\vec{r}), \tag{5}$$

where

$$W_i(\vec{r}) = e^2 \int \mathrm{d}\vec{y}\, [Z_i^2 n_i^2(\vec{y}) + Z_1 Z_2 n_1(\vec{y}) n_2(\vec{y})]/(|\vec{r} - \vec{y}| n_i(\vec{y})), \tag{6}$$

and μ_i are the chemical potentials, which are related to the ground-state energy, Eq. (3), by the general thermodynamics identity

$$\mu_i = \frac{\partial E}{\partial N_i}.$$

We note that the mean-field theory, Eq. (5), cannot describe the Wigner-crystallization regime.[11]

In the Thomas–Fermi (TF) approximation, in which one neglects the kinetic energy terms in Eq. (5), Eq. (5) reduce to

$$\mu_i = V_i + W_i. \tag{7}$$

Equation (7) holds in the region where n_i are positive and $n_i = 0$ outside this region. We can obtain from these equations that

$$\mu_2 - \frac{Z_2}{Z_1}\mu_1 = \left(\frac{m_2\omega_2^2}{m_1\omega_1^2} - \frac{Z_2}{Z_1}\right)\frac{m_1\omega_1^2}{2}r^2,$$

and hence we have proved that Eq. (7) has non-trivial solution if and only if

$$\lambda = \frac{m_2\omega_2^2 Z_1}{m_1\omega_1^2 Z_2} = 1. \tag{8}$$

In this case, we have $\mu_2 = (Z_2/Z_1)\mu_1$.

Equation (7) can be solved analytically to obtain

$$n_i(\vec{r}) = \frac{3N_i}{4\pi R_i^3}\theta(R_i^2 - r^2), \tag{9}$$

where θ denotes the unit positive step function,

$$R_i = \sqrt{\frac{\hbar}{m_i\omega_i}}[\gamma_c^{(i)}(Z_i^2 N_i^2 + Z_1 Z_2 N_1 N_2)/N_i]^{1/3}, \tag{10}$$

and

$$\gamma_c^{(i)} = \alpha\sqrt{\frac{m_i c^2}{\hbar\omega_i}}.$$

This is done by seeing that potentials W_i, Eq. (6) are solutions of the Poisson equations

$$\nabla^2 W_1(\vec{r}) = -4\pi e^2[Z_1^2 n_1(\vec{r}) + Z_1 Z_2 n_2(\vec{r})],$$

$$\nabla^2 W_2(\vec{r}) = -4\pi e^2[Z_2^2 n_2(\vec{r}) + Z_1 Z_2 n_1(\vec{r})].$$

Straightforward calculations with n_i from Eq. (9) yield

$$\mu_i = \frac{3}{2} m_i \omega_i^2 R_i^2,$$

and

$$E = \frac{9}{10}\frac{\hbar\omega_1}{Z_1^2}[\gamma_c^{(1)}]^{2/3}[(Z_1^2 N_1 + Z_1 Z_2 N_2)]^{5/3}.$$

Comparing the radii of clouds R_1 and R_2, Eq. (10), we see that $R_1 = R_2$. Therefore, we have found that depending of the ratio λ, Eq. (8), the two components coexist in the same region of space, in spite of the Coulomb repulsion between the two species. This result is obtained in the TF approximation, Eq. (7). If $\lambda = 1$, and $N_i \gg 1$, $\gamma_c^{(i)} N_j \gg 1$, the TF approximation provides an accurate description of the exact mean-field solution (except a narrow region near a surface).

For a general value of λ, the mixture becomes unstable against deviations from uniformity. Although the TF approximation is not applicable for this case, we expect that if $\lambda \approx 1$ and $N_i \gg 1$, $\gamma_c^{(i)} N_j \gg 1$, the two components may coexist in the same regions of space.

4. Fusion Rates

For the two species case, we generalize the one-specie Fermi pseudo-potential[3] as

$$\mathrm{Im}\, V_{ij}^{\mathrm{F}}(\vec{r}) = -\frac{A_{ij}\hbar\delta(\vec{r})}{2},$$

where the nuclear reaction rate constants A_{ij} are given by

$$A_{ij} = \frac{2 S_{ij} r_{\mathrm{B}}^{(ij)}}{\pi\hbar}$$

with $r_{\mathrm{B}}^{(ij)} = \hbar^2/2\mu_{ij} Z_i Z_j e^2$ and $\mu_{ij} = m_i m_j/(m_i + m_j)$. S_{ij} are the S-factors for nuclear fusion between two nuclei from species i and j.

The nucleus–nucleus fusion rates are determined from the trapped ground state wave function Ψ as

$$R_{11} = -\frac{2}{\hbar}\sum_{i<j}^{N_1}\langle\Psi|\mathrm{Im}\, V_{11}^{\mathrm{F}}(\vec{x}_i - \vec{x}_j)|\Psi\rangle/\langle\Psi|\Psi\rangle,$$

$$R_{12} = -\frac{2}{\hbar}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\langle\Psi|\mathrm{Im}\, V_{12}^{\mathrm{F}}(\vec{x}_i - \vec{y}_j)|\Psi\rangle/\langle\Psi|\Psi\rangle,$$

$$R_{22} = -\frac{2}{\hbar}\sum_{i<j}^{N_2}\langle\Psi|\mathrm{Im}\,V_{22}^{\mathrm{F}}(\vec{y}_i - \vec{y}_j)|\Psi\rangle/\langle\Psi|\Psi\rangle,$$

and in the mean-field approximation, Eq. (9), we have

$$R_{11} = A_{11}N_1 n_1^{\mathrm{B}}/2, \qquad R_{12} = A_{12}N_1 n_2^{\mathrm{B}}, \qquad R_{22} = A_{22}N_2 n_2^{\mathrm{B}}/2,$$

where

$$n_i^{\mathrm{B}} = N_i/(4/3)\pi R_i^3.$$

If the probabilities of the mean-field ground state occupation,[3] Ω_i are taken into account, the trap fusion rates are given by

$$R_{11}^{\mathrm{t}} = \Omega_1 R_{11}, \qquad R_{22}^{\mathrm{t}} = \Omega_2 R_{22}, \qquad R_{12}^{\mathrm{t}} = \Omega_3 R_{12}.$$

We expect that $\Omega_3 \approx \sqrt{\Omega_1\Omega_2}$.

5. Selection Rules

For the BEC mechanism for LENR and transmutation processes, there are two selection rules: (A) nuclear spin selection rule and (B) nuclear mass-charge selection rule.

(A) Nuclear spin selection rule:

Nuclear spins of both species must be integer. This rule is obvious for the BEC mechanism.

(B) Nuclear mass–charge selection rule:

If we assume $\omega_1 = \omega_2$, we have from Eq. (8), $\lambda = m_2 Z_1/m_1 Z_2 = 1$ or

$$\frac{Z_1}{Z_2} = \frac{m_1}{m_2} \approx \frac{Z_1 + \tilde{N}_1}{Z_2 + \tilde{N}_2}, \tag{11}$$

where $\tilde{N}_i$ is the number of neutrons in the Bose nucleus for the specie i. We note that Eq. (11) is satisfied, for example, for two species with $Z_i = \tilde{N}_i$.

6. Application to (D + Li) Reactions

For (d + ^{6}Li) reaction, ^{6}Li(d,α)^{4}He ($Q = 22.37$ MeV) and for (d + ^{7}Li) reaction, ^{7}Li(d,n)2 ^{4}He ($Q = 15.12$ MeV), the S-factors are 18.8 MeV-barn[12,13] and 30 MeV-barn,[14] respectively. Using these values we find that the corresponding nuclear reaction rate constants are $A_{\mathrm{d}^6\mathrm{Li}} \approx 5.8 \times 10^{-15}\mathrm{cm}^3/\mathrm{s}$ and $A_{\mathrm{d}^7\mathrm{Li}} \approx 8.97 \times 10^{-15}\mathrm{cm}^3/\mathrm{s}$ which are about 50 times larger than the d–d nuclear reaction rate constant $A_{\mathrm{dd}} \approx 1.5 \times 10^{-16}\mathrm{cm}^3/\mathrm{s}$.

We expect that nuclear reaction rate constants for reactions ^{6}Li(^{6}Li,^{5}Li)^{7}Li ($Q = 1.86$ MeV) and ^{6}Li(^{6}Li,α)2 ^{4}He ($Q = 20.897$ MeV) are much smaller than $A_{\mathrm{d}^6\mathrm{Li}}$.

For the BEC mechanism, the (d + ^{7}Li) reaction rate is expected to be suppressed compared with the (d + ^{6}Li) reaction rate. This is consistent with the observation of depletion of ^{6}Li[15,16] as inferred from increased ^{7}Li/^{6}Li abundance ratio in Arata–Zhang's particulate Pd exposed to deuterium gas.[17–20]

The excess heat and ^{4}He observed in electrolysis experiments[17–20] may be due to ^{6}Li(d,α)^{4}He in addition to other reactions leading to the final states without ^{4}He. This would be an alternative scenario to the (d + d) reaction scenario which has been proposed by many.

7. Summary and Conclusions

A generalization of the BEC mechanism for one specie LENR processes in condensed matters has been made to the case of a mixture of two different species of positively charged Bose nuclei in harmonic traps. Depending on the ratio of the parameters involved, it is shown that the two components may coexist in same regions of space, in spite of the Coulomb repulsion between two species. We have obtained an approximate selection rule involving nuclear masses and charges of two species.

For a mixture d and Li species, we expect that the (d + ^{6}Li) reaction rate may be larger than the (d + d) reaction rate, implying that the (d + ^{6}Li) reactions may dominate over the (d + d) reactions in LENR experiments in condensed matters. This is consistent with the recent observation of the ^{6}Li depletion[15,16] in particulate Pd exposed to deuterium gas.[17–20] Further LENR experiments involving ^{6}Li or ^{7}Li separately are needed for more conclusive tests of the BEC mechanism with two species.

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